Expanding Universe: slowdown or speedup?

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The purpose of the review - to provide affordable to a wider audience a description of kinematics of the cosmological expansion, and dynamic interpretation of this process. The focus will be the accelerated expansion of the Universe. Will be considered by virtually all major opportunity to explain the accelerated expansion of the Universe, including an introduction to the energetic budget of the universe of so-called dark energy, a modification of Einstein's equations, and a new approach based on the holographic principle.

I. INTRODUCTION

There are two fundamental reasons making the question put in the title so principal. Firstly, in common sense, the correct answer is necessary (though insufficient) to predict the ultimate fate of the Universe. To be more specific, one must know current values of the kinematic parameters (velocity, acceleration etc.) in order to fix initial conditions necessary for solution of the differential equations describing dynamics of the Universe. Secondly, description of either decelerated or accelerated expansion of the Universe requires absolutely distinct cosmological models. Conventional substances (non-relativistic matter and radiation) from one hand and general relativity from the other explain the decelerated expansion. In order to obtain the accelerated expansion one must either change the composition of the Universe rather radically, or make even more responsible conclusion: the fundamental physical theories lying in the basis of our understanding of the World, are essentially wrong. Common experience tells us that it is very unlikely to answer the posed question in near future. Already 80 years passed after the discovery of the Universe expansion, and we still cannot establish the source responsible for the initial velocity distribution. "The Big Bang" is nothing but euphemism used to hide limitation of our knowledge. What does actually expand? This question is still sharply controversial: "How can vacuum expand?" [1]. Even more miraculous, how can vacuum expand with acceleration or deceleration? "...how is it possible for space, which is utterly empty, to expand? How can nothing expand? The answer is: space does not expand. Cosmologists sometimes talk about expanding space, but they should know better." [2].

No generation of homo sapiens can vanquish the temptation to believe that it is chosen to acquire complete and final understanding of Nature and Universe. In particular, the first half of the last century, with discovery of Universe expansion by Hubble and due to now fundamental physical theories, relativity and quantum mechanics first of all, was marked by new cosmological paradigm — the Big Bang model. The main basis of the model is the celebrated Hubble law, discovered in twenties of the last century and supported by all types of up-to-date cosmological observations. The Big Bang model was able to give satisfactory explanation for thermal evolution of cooling Universe, predicted existence of relict radiation, correctly described relative abundance of light elements and many other features of the Universe.

Towards the end of the last century it was commonly hoped that the Big Bang model supplemented by the Inflation Theory represented an adequate (at least as a first approximation) model of Universe. However this hope was never to come true. The cosmological paradigm had to change for the reason of observations performed with ever increasing precision. Right after the Hubble discovery the cosmologists tempted to measure the deceleration of the expansion, attributed to gravitation. They were so firmly confident to discover exactly above stated effect, that the corresponding observable was named the deceleration parameter. However, in the year 1998 two independent collaborations [3, 4], studying distant supernovae, presented convincing evidence for the fact that the Universe expansion is accelerated. It turned out that the brightness decreases in average much faster than it was commonly believed according to the Big Bang model. Such additional dimming means that the distance, attributed to given redshift, was somewhat underestimated. And it in turn means that the universe expansion accelerates: in the past the Universe expanded slower than nowadays. The discovery of cosmological acceleration is probably one of the most important observations not only in

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modern cosmology but in physics in general. Accelerated expansion of Universe the most directly demonstrates the fact that our fundamental theories are either incomplete, or, even worse, misleading [5?].

Physical origin of the cosmological acceleration still remains greatest miracle. As we already mentioned above, if the Universe is solely composed of matter and radiation, it must decelerate the expansion. If the expansion in fact accelerates, we have to choices, each of which forces us to revise our basic physical concepts.

1. Up to 75% of energy density in the Universe exists in form of unknown substance (commonly called the dark energy) with high negative pressure, providing the accelerated expansion.

2. General Relativity theory must be revised on cosmological scales.

We should remark that, besides the two above cited radical possibilities to put the theory in agreement with the observations, there still exist an obvious conservative way to get rid of the problem: it implies more adequate utilization of available theoretical possibilities. D.Wiltshire [12] expressed this idea in the following way: I will take the viewpoint that rather than adding further epicycles to the gravitational action [1] the cosmological observations which we currently interpret in terms of dark energy are inviting us to think more deeply about the foundations of general relativity. The above mentioned term "more adequate utilization of the available possibilities" need an explanation. Let us give an example. In the context of cosmological models based on homogeneous and isotropic Universe, in order to explain the observed acceleration one have to involve a new form of matter with negative pressure — the dark energy. According to an alternative interpretation [12–15], the accelerated expansion of the Universe follows from deviations from the homogeneity. It is assumed that mass density in the Universe is considerably inhomogeneous on the scales smaller than the Hubble radius. In order to transit to effective description of homogeneous and isotropic Universe, one should average and/or smooth out the inhomogeneities up to some properly chosen scale of the averaging. In such an averaged Universe one can define the "effective cosmological parameters". After that it turns out that so obtained equations of motion in general differ from the equations with the same parameters in the models based on the cosmological principle of uniform and isotropic Universe. If the difference looks like an effective dark energy contribution, then one can try to explain the accelerated expansion of Universe in frames of General Relativity without any dark energy.

The present review considers all the three above listed possibilities. It aims to present a comprehensive description for both the kinematics of the cosmological expansion and the dynamical description of the process. We concentrate our attention on the acceleration of the cosmological expansion. It should be emphasized that the equivalence principle directly links the acceleration with the gravity nature and space-time geometry. Therefore its value is extremely important for verification of different cosmological models. It is the quantity which to great extent determines the ultimate fate of the Universe.

II. COSMOGRAPHY — THE KINEMATICS OF EXPANDING UNIVERSE

This section is devoted to a particular way of Universe description, called the "cosmography"[16], entirely based on the cosmological principle. The latter states that the Universe is homogeneous and isotropic on the scales larger than hundreds megaparsecs, and it allows, among the diversity of models for description of the Universe, to select a specific class of uniform and isotropic ones. The most general space-time metrics consistent with the cosmological principle is the Friedmann-Robertson-Walker (FRW) one:

$$ds^2 = dt^2 - a^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right\}. \tag{1}$$

Here $a(t)$ is the scale factor and r is coordinate of a point, which does not take part in any motion except the global expansion of the Universe. In some sense the main problem of cosmology is to determine the dependence $a(t)$.

The cosmological principle helps to construct the metrics of the Universe and to make the first steps in interpretation of cosmological observations. Recall that kinematics means the part of mechanics that describes motion of bodies irrelative to forces causing it. In that sense the cosmography is nothing but kinematics of cosmological expansion. In order to construct the principal cosmological quantity — the time dependence of the scale factor $a(t)$ — one have to have the equations of motion (the Einstein equations of general relativity) and an assumption on material composition of the Universe, allowing to construct the energy-momentum tensor. The efficiency of cosmography lies in the fact

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[1] generally speaking, "epicycles" here stand not only for additional terms in the Hilbert-Einstein action, but also for such new substances as dark energy and dark matter

that it allows to verify arbitrary cosmological models obeying the cosmological principle. Modifications of general relativity or introduction of new components (dark matter or dark energy) will certainly change the dependence $a(t)$, but will not affect the relations between the kinematic characteristics. Velocity of the Universe expansion, determined by the Hubble parameter $H(t) \equiv \dot{a}(t)/a(t)$, depends on time. The time dependence of the latter is measured by the deceleration parameter $q(t)$. Let us define it by Taylor expansion of the time dependence of the scale factor $a(t)$ in vicinity of the current moment of time t_0:

$$a(t) = a(t_0) + \dot{a}(t_0)[t - t_0] + \frac{1}{2}\ddot{a}(t_0)[t - t_0]^2 + \cdots \tag{2}$$

Let us rewrite the relation in the following form

$$\frac{a(t)}{a(t_0)} = 1 + H_0[t - t_0] - \frac{q_0}{2}H_0^2[t - t_0]^2 + \cdots, \tag{3}$$

where the deceleration parameter reads

$$q(t) \equiv -\frac{\ddot{a}(t)a(t)}{\dot{a}^2(t)} = -\frac{\ddot{a}(t)}{a(t)}\frac{1}{H^2(t)}. \tag{4}$$

As will be shown below, accelerated growth of the scale factor occurs at $q < 0$, while accelerated growth of the expansion velocity $\dot{H} > 0$ corresponds to $q < -1$. When the sign of the deceleration parameter was first defined, it seemed evident that gravitation — the only force governing the dynamics of Universe — dumps its expansion. A natural desire to deal with a positive parameter predetermined the choice of the sign. Afterwards it turned out that the choice made is inconsistent with the observed dynamics of expansion and it is rather an example of historical curiosity.

For more detailed kinematical description of cosmological expansion it is useful to consider the extended parameter set [17, 18]

$$\begin{aligned}
H(t) &\equiv \frac{1}{a}\frac{da}{dt};\\
q(t) &\equiv -\frac{1}{a}\frac{d^2a}{dt^2}\left[\frac{1}{a}\frac{da}{dt}\right]^{-2};\\
j(t) &\equiv \frac{1}{a}\frac{d^3a}{dt^3}\left[\frac{1}{a}\frac{da}{dt}\right]^{-3};\\
s(t) &\equiv \frac{1}{a}\frac{d^4a}{dt^4}\left[\frac{1}{a}\frac{da}{dt}\right]^{-4};\\
l(t) &\equiv \frac{1}{a}\frac{d^5a}{dt^5}\left[\frac{1}{a}\frac{da}{dt}\right]^{-5}.
\end{aligned}$$

Note that the latter four parameters are dimensionless. Derivatives of lower parameters can be expressed through higher ones. So, for example,

$$\frac{dq}{d\ln(1+z)} = j - q(2q+1).$$

Let us Taylor expand the time dependence of the scale factor using the above introduced parameters

$$\begin{gathered}
a(t) = a_0\,[\,1 + H_0(t - t_0) - \tfrac{1}{2}q_0H_0^2(t - t_0)^2 + \\
\tfrac{1}{3!}j_0H_0^3(t - t_0)^3 + \tfrac{1}{4!}s_0H_0^4(t - t_0)^4 + \tfrac{1}{5!}l_0H_0^5(t - t_0)^5 \;+ \mathrm{O}\left((t - t_0)^6\right)\,].
\end{gathered} \tag{5}$$

In terms of the same parameters the Taylor series for redshift reads

$$1 + z = \left[\begin{array}{c} 1 + H_0(t - t_0) - \frac{1}{2}q_0H_0^2(t - t_0)^2 + \frac{1}{3!}j_0H_0^3(t - t_0)^3 + \frac{1}{4!}s_0H_0^4(t - t_0)^4 \\ + \frac{1}{5!}l_0H_0^5(t - t_0)^5 \;+ \mathrm{O}\left((t - t_0)^6\right) \end{array}\right]^{-1}; \tag{6}$$

$$z = H_0(t_0 - t) + \left(1 + \frac{q_0}{2}\right)H_0^2(t - t_0)^2 + \cdots. \tag{7}$$

Let us cite few useful relations for the deceleration parameter

$$\begin{aligned}
q(t) &= \frac{d}{dt}\left(\frac{1}{H}\right) - 1; \\
q(z) &= \frac{1+z}{H}\frac{dH}{dz} - 1; \\
q(z) &= \frac{d\ln H}{dz}(1+z) - 1; \\
q(a) &= -\left(1 + \frac{\frac{dH}{dt}}{H^2}\right) = -\left(1 + \frac{a\frac{dH}{da}}{H}\right); \\
q &= -\frac{d\ln(aH)}{d\ln a}.
\end{aligned}$$

For the case of single component liquid with density ρ one has

$$q(a) = -1 - \frac{a\frac{d\rho}{da}}{2\rho}. \tag{8}$$

The derivatives $\frac{dH}{dz}$, $\frac{d^2H}{dz^2}$, $\frac{d^3H}{dz^3}$ and $\frac{d^4H}{dz^4}$ can be expressed through the parameters q and j solely

$$\begin{aligned}
\frac{dH}{dz} &= \frac{1+q}{1+z}H; \\
\frac{d^2H}{dz^2} &= \frac{j-q^2}{(1+z)^2}H; \\
\frac{d^3H}{dz^3} &= \frac{H}{(1+z)^3}\left(3q^2 + 3q^3 - 4qj - 3j - s\right); \\
\frac{d^4H}{dz^4} &= \frac{H}{(1+z)^4}\left(-12q^2 - 24q^3 - 15q^4 + 32qj + 25q^2 j + 7qs + 12j - 4j^2 + 8s + l\right).
\end{aligned}$$

Let us present for convenience the useful relations for transformations from higher time derivatives to derivatives with respect to red shift:

$$\frac{d^2}{dt^2} = (1+z)H\left[H + (1+z)\frac{dH}{dz}\right]\frac{d}{dz} + (1+z)^2H^2\frac{d^2}{dz^2}; \tag{9}$$

$$\begin{aligned}
\frac{d^3}{dt^3} &= -(1+z)H\left\{H^2 + (1+z)^2\left(\frac{dH}{dz}\right)^2 + (1+z)H\left[4\frac{dH}{dz} + (1+z)\frac{d^2H}{dz^2}\right]\right\}\frac{d}{dz} - 3(1+z)^2H^2 \\
&\times \left[H + (1+z)\frac{dH}{dz}\right]\frac{d^2}{dz^2} - (1+z)^3H^3\frac{d^3}{dz^3};
\end{aligned} \tag{10}$$

$$\begin{aligned}
\frac{d^4}{dt^4} &= (1+z)H\left[H^2 + 11(1+z)H^2\frac{dH}{dz} + 11(1+z)H\frac{dH}{dz} + (1+z)^3\left(\frac{dH}{dz}\right)^3 + 7(1+z)^2H\frac{d^2H}{dz^2}\right. \\
&+ \left.4(1+z)^3H\frac{dH}{dz}\frac{d^2H}{d^2z} + (1+z)^3H^2\frac{d^3H}{d^3z}\right]\frac{d}{dz} + (1+z)^2H^2\left[7H^2 + 22H\frac{dH}{dz} + 7(1+z)^2\left(\frac{dH}{dz}\right)^2\right. \\
&+ \left.4H\frac{d^2H}{dz^2}\right]\frac{d^2}{dz^2} + 6(1+z)^3H^3\left[H + (1+z)\frac{dH}{dz}\right]\frac{d^3}{dz^3} + (1+z)^4H^4\frac{d^4}{dz^4} + (1+z)^4H^4\frac{d^4}{dz^4};
\end{aligned} \tag{11}$$

Derivatives of the Hubble parameter squared with respect to the red shift $\frac{d^{(i)}H^2}{dz^{(i)}}$, $i = 1,2,3,4$ expressed through the cosmographic parameters take the form

$$\frac{d(H^2)}{dz} = \frac{2H^2}{1+z}(1+q)$$

$$\frac{d^2(H^2)}{dz^2} = \frac{2H^2}{(1+z)^2}(1+2q+j);$$

$$\frac{d^3(H^2)}{dz^3} = \frac{2H^2}{(1+z)^3}(-qj-s);$$

$$\frac{d^4(H^2)}{dz^4} = \frac{2H^2}{(1+z)^4}(4qj+3qs+3q^2j-j^2+4s+l).$$

Time derivatives of the Hubble parameter can be expressed through above defined cosmographic parameters H, q, j, s, l as

$$\begin{aligned} \dot{H} &= -H^2(1+q); \\ \ddot{H} &= H^3\left(j+3q+2\right); \\ \dddot{H} &= H^4\left[s-4j-3q(q+4)-6\right]; \\ \ddddot{H} &= H^5\left[l-5s+10\left(q+2\right)j+30(q+2)q+24\right]. \end{aligned} \tag{12}$$

From the relations (12) one can easily see that the accelerated growth of the acceleration velocity $\dot{H} > 0$ corresponds to the case $q < -1$.

The Hubble parameter, as can be seen from the relation (8), is linked to the deceleration parameter with the integral relation

$$H = H_0 \exp\left[\int_0^z \left[q(z')+1\right] d\ln(1+z')\right].$$

It immediately follows that in order to calculate the principal characteristic of the expanding Universe $H(z)$ one needs the information on dynamics of cosmological expansion, coded in the quantity $q(z)$.

III. DYNAMICS OF COSMOLOGICAL EXPANSION BRIEFLY

Dynamics of Universe in frames of General Relativity is described by the Einstein equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}.$$

The energy-momentum tensor $T_{\mu\nu}$ describes the space distribution of mass(energy), while the curvature tensor components $R_{\mu\nu}$ and its trace R are expressed through the metric tensor $g_{\mu\nu}$ and its derivatives of first and second order. The Einstein equations generally are non-linear and hard to analyze. The problem is simplified when considering mass distribution with special symmetry properties provided by the metric. For homogeneous and isotropic Universe described by the FRW-metric, the Einstein equations reduce to the system of Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \tag{13}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right). \tag{14}$$

Here ρ and p are total energy density and pressure respectively for all the components present in the Universe at the considered time moment.

The system (13,14) is sufficient for complete description of the Universe dynamics. The conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{15}$$

follows from the Lorentz-invariance of the energy-momentum tensor $T^{\mu}_{\nu,\mu} = 0$ and, as can be easily seen, it is nothing but the first principle of thermodynamics for ideal liquid with constant entropy $dE + pdV = 0$. We remark, that this equation can be derived from the Friedman equations (13,14).

Let us now consider relative density for ith component of the Universe:

$$\Omega_i \equiv \frac{\rho_i}{\rho_c}, \quad \rho_c \equiv \frac{3H^2}{8\pi G}, \quad \Omega \equiv \sum_i \Omega_i. \tag{16}$$

Setting the relative curvature density by definition $\Omega_k \equiv -\frac{k}{a^2H^2}$, the first Friedmann equation can be presented in the form

$$\sum_i \Omega_i = 1.$$

In order to solve the Friedmann equations one has to define the matter composition of the Universe and construct the state equation for each component. In the simplest linear form the state equation reads

$$p_i = w_i \rho_i. \tag{17}$$

Solving the Friedmann equations for the case $w = const$, $k = 0$ one obtains

$$a(t) \propto (t/t_0)^{\frac{2}{3(1+w)}}, \ \rho \propto a^{-3(1+w)}.$$

(The scale factor is normalized by the condition $a\,(t_0) = 1$.) The above cited solutions exist only provided $w \neq -1$, the otherwise case will be considered separately. For the Universe dominated by radiation (relativistic gas of photons and neutrinos) one has $w = 1/3$, while for the matter dominated case $w = 0$. As a result of such state equations, for the matter dominated case one gets

$$a(t) \propto (t/t_0)^{2/3}, \ \rho \propto a^{-3}.$$

which is easily interpreted as the particle number conservation law. For the radiation-dominated case the solution reads

$$a(t) \propto (t/t_0)^{1/2}, \quad \rho \propto a^{-4}.$$

The latter result is a consequence of the fact that the energy density of radiation decreases as a^{-3} due to growth of occupied volume (as Universe expands) and additionally as a^{-1} due to the redshift. Note that from the equation (15) it follows that $\rho = const$ for $w = -1$. In the latter case the hubble parameter is constant leading to exponential growth of the scale factor

$$a(t) \propto e^{Ht}.$$

Therefore in the case of traditional cosmological components in form of matter and radiation with $w = 0$ and $w = 1/3$ respectively, the Universe expansion can only decelerate, i.e. $\ddot{a} < 0$.

Using the definition of deceleration parameter, one can see that for flat Universe filled by single component with state equation $p = w\rho$

$$q = \frac{1}{2}(1 + 3w).$$

In generic case ($k = (0, \pm 1)$, $\quad \rho = \sum\limits_i \rho_i, \quad p = \sum\limits_i \rho_i w_i$) one gets

$$q = \frac{\Omega}{2} + \frac{3}{2}\sum_i w_i \Omega_i. \tag{18}$$

Using (16), the latter relation can be rewritten in the form

$$q = \frac{1}{2}\left(1 + \frac{k}{a^2H^2}\right)\left(1 + 3\frac{p}{\rho}\right).$$

Since the deceleration parameter q is a slowly varying quantity (e.g. $q = 1/2$ for matter-dominated case and $q = -1$ in the Universe dominated by dark energy in form of cosmological constant), then the useful information is contained

in its time average value, which is very interesting to obtain without integration of the equations of motions for the scale factor. Let us see how it is possible [21]. Let us define the time average $\bar{q}$ on the interval $[0, t_0]$ by the expression

$$\bar{q}\,(t_0) = \frac{1}{t_0}\int_0^{t_0} q(t)dt.$$

Substituting the definition of the deceleration parameter

$$q(t) = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1,$$

it is easy to see that

$$\bar{q}(t_0) = -1 + \frac{1}{t_0 H_0} \tag{19}$$

or, equivalently,

$$t_0 = \frac{H_0^{-1}}{1+\bar{q}}. \tag{20}$$

As was expected, the present age of universe scales as H_0^{-1}, but the proportionality coefficient depends only on the average value of the deceleration parameter. It is worth to mention that this purely kinematic result does not dependent on curvature of space in the Universe, neither on number of component filling it, nor on the particular type of the gravity theory. Let us somewhat reformulate the obtained results for the average deceleration parameter. For single-component flat Universe the Friedman equations can be presented in the following form

$$8\pi G\rho \;=\; 3\frac{\dot{a}^2}{a^2}; \tag{21}$$

$$8\pi G p \;=\; -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}. \tag{22}$$

For the single component Universe with the state equation $p = w\rho, \quad w = const$ the equation for the scale factor

$$a\ddot{a} + \left(\frac{1+3w}{2}\right)\dot{a}^2 = 0,$$

has an exact solution

$$a(t) = a_0\left[\frac{3}{2}\,(1+w)\,H_0 t\right]^{\frac{2}{3(1+w)}}. \tag{23}$$

Now it immediately follows that

$$q = \frac{1+3w}{2} = const, \quad t_0 = \frac{2H_0^{-1}}{3(1+w)}. \tag{24}$$

In particular, one has for the cases of radiation $(w = 1/3)\,,\ q = 1$ and non-relativistic matted $(w = 0)\,,\ q = 1/2$. From the other hand, accounting that in the considered case $q = \bar{q}$, the latter relation can be rewritten as $t_0 = H_0^{-1}/\left(1+\bar{q}\right)$, which coincides with the relation (20). The resulting expression (24) can be presented in the following form

$$T = \frac{H^{-1}}{1+\bar{q}}, \tag{25}$$

where $T, H, \bar{q}$ are age of Universe, Hubble parameter and average deceleration parameter respectively. Since $\bar{q}$ is of order of unity, then it immediately follows from (23) that on any stage of Universe evolution the Hubble time H_0^{-1} represents the characteristic time scale.

Further model-independent dynamical restrictions on the Universe kinematics can be derived from the so-called energy conditions [22, 24–27]. Those conditions based on rather general physical principles, impose restrictions on the energy-momentum tensor components $T_{\mu\nu}$. Specifying a particular model for the medium[2] those conditions can

[2] specific model does not specify the state equation!

be transformed into inequalities, limiting possible values of pressure and density in the medium. In the Friedmann model the medium is an ideal fluid, where

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p g_{\mu\nu}, \tag{26}$$

where u_μ is the fluid four-velocity, with total density ρ and pressure p given, respectively, by

$$\rho = \frac{3}{8\pi G}\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right), \quad p = -\frac{1}{8\pi G}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right). \tag{27}$$

The most common energy conditions (see, e.g., [22, 24–26]) reduce to

$$\begin{aligned}
NEC &\;\Rightarrow\; \rho + p \geq 0;\\
WEC &\;\Rightarrow\; \rho \geq 0 \quad and \quad \rho + p \geq 0;\\
SEC &\;\Rightarrow\; \rho + 3p \geq 0, and \quad \rho + p \geq 0;\\
DEC &\;\Rightarrow\; \rho \geq 0 \quad and \quad -\rho \leq p \leq \rho,
\end{aligned}$$

where NEC, WEC, SEC and DEC correspond, respectively, to the null, weak, strong and dominant energy conditions. Because these conditions do not require a specific equation of state of the matter in the Universe, they provide very simple and model-independent bounds on the behavior of the energy density and pressure. Therefore, the energy conditions are among of many approaches to understand the evolution of Universe. From Eqs. (27), one easily obtains that these energy conditions can be translated into the following set of dynamical constraints relating the scale factor

$$\begin{aligned}
NEC &\;\Rightarrow\; -\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \geq 0;\\
WEC &\;\Rightarrow\; \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \geq 0;\\
SEC &\;\Rightarrow\; \frac{\ddot{a}}{a} \leq 0;\\
DEC &\;\Rightarrow\; \frac{\ddot{a}}{a} + 2\left[\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right] \geq 0.
\end{aligned} \tag{28}$$

For flat Universe the conditions (28) can be transformed to restrictions on the deceleration parameter q

$$\begin{aligned}
NEC &\;\Rightarrow\; q \geq -1;\\
SEC &\;\Rightarrow\; q \geq 0;\\
DEC &\;\Rightarrow\; q \leq 2,
\end{aligned} \tag{29}$$

while WEC is always satisfied for arbitrary real $a(t)$.

The conditions (29), considered separately, leave a principal possibility for both decelerated ($q > 0$) and accelerated ($q < 0$) expansion of Universe. The sense of the restrictions for NEC in (29) is quite clear. As it follows from the second Friedmann equation, the condition of accelerated expansion of Universe reduce to the inequality $\rho + 3p \leq 0$, i.e. accelerated expansion of Universe is possible only in presence of components with high negative pressure $p < -1/3\rho$. The energetic condition SEC excludes existence of such components. As a result $q \geq 0$ in that case. At the same time the conditions NEC and DEC are compatible with the condition $p < -1/3\rho$ and therefore regimes with $q < 0$ are allowed here. In conclusion of the section let us pay attention to an interesting feature of the expanding Universe dynamics. According to Hubble law, the galaxies situated on the Hubble sphere, recede from us with light speed. Velocity of the Hubble sphere itself equals to time derivative of the Hubble radius $R_H = c/H$,

$$\frac{d}{dt}(R_H) = c\frac{d}{dt}\left(\frac{1}{H}\right) = -\frac{c}{H^2}\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right) = c(1+q). \tag{30}$$

In the Universe with decelerated expansion ($q > 0$) the Hubble sphere has superluminal velocity $c(q+1)$ and it outruns those galaxies. Therefore the galaxies initially situated outside the Hubble sphere, gradually enter inside it. Any observer in arbitrary point of the Universe will see that the number of galaxies constantly increases. In the case of accelerated expanding Universe ($q < 0$) the Hubble sphere has subluminal $c(1 - |q|)$ and it lags behind the galaxies. Therefore now the galaxies initially contained in the Hubble sphere will gradually leave it and become unaccessible for observation. Should we consider those unobservable galaxies as physically real? The distinction between physics

and metaphysics is the possibility of experimental verification for the physical theories. Physics have no deal with unobservable objects. However, the science constantly expands its boundaries, thus including to consideration more and more abstract concepts, once been metaphysical before: atoms, electromagnetic waves, black holes... The list can be continued further.

We are most likely inhabitants of accelerated expanding Universe. Likewise in the decelerated expanding one, there are galaxies so distant from us that no signal from them can be detected by terrestrial observer. However, if the cosmic expansion accelerates, then we recede from those galaxies with superluminal velocity. Therefore, if their light have not reach us till now then it will never come. Such galaxies are unaccessible for observation not only temporarily, they are forever unobservable. Such "never observable galaxies" descent from the same "Big Bang" like our Milky Way. Should we attribute them to physical objects or to metaphysics? Those who consider the science fiction as a realization of most unbounded fantasy, are absolutely wrong. Compared to modern cosmology, the science fiction is dull and lacks imagination.

Accelerated expansion of the Universe was first introduced in cosmological models with creation of the inflation theory. It was developed to fix multiple defeats of the Big Bang model. It turns out that in order to resolve most of them it is sufficient to have exponentially fast accelerated expansion of Universe at the very beginning of its evolution, during just about $10^{-35}s$. The simplest way to obtain expansion of such type is to consider dynamics of Universe with scalar field. The inflation theory is formulated in many ways, starting from the models based on quantum gravity and high-temperature phase transitions theory with supercooling and exponential expansion in the false vacuum state. To illustrate the main ideas of the theory let us consider flat, homogeneous and isotropic Universe filled by the scalar field φ with the potential $V(\varphi)$, independent of the coordinates. The first Friedmann equation (13) in the considered case takes the following form

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right).$$

The conservation equation (15), written in terms of the scalar field, takes the form of Klein-Gordon equation, and in the case of non-stationary background one obtains:

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0. \tag{31}$$

In the fast expanding Universe the scalar field rolls down very slowly, like a ball in viscous medium, while the effective viscosity turns out to be proportional to the expansion velocity. In the slow-roll regime

$$H\dot{\varphi} \gg \ddot{\varphi}, \quad V(\varphi) \gg \dot{\varphi}^2.$$

In the same limit the equations of motion take the form

$$3H\dot{\varphi} + V'(\varphi) = 0;$$
$$H^2 = \frac{8\pi}{3M_{Pl}^2}V(\varphi).$$

To be more specific, let us consider the simplest model of the scalar field with mass m and potential energy $V(\varphi) = \frac{m^2}{2}\varphi^2$. Right after the start of inflation $\ddot{\varphi} \ll 3H\dot{\varphi}$; $\dot{\varphi}^2 \ll m^2\varphi^2$, therefore

$$3\frac{\dot{a}}{a}\dot{\varphi} + m^2\varphi = 0;$$
$$H = \frac{\dot{a}}{a} = \frac{2m\varphi}{M_{Pl}}\sqrt{\frac{\pi}{3}}.$$

Due to fast growth of the scale factor and slow variation of the field because of strong friction, one has

$$a \propto e^{Ht}, \quad H = \frac{2m\varphi}{M_{Pl}}\sqrt{\frac{\pi}{3}}.$$

For better clarity let us obtain the state equation for the scalar field in the slow-roll regime. For homogeneous scalar field in the potential $V(\varphi)$ in the local Lorentz frame the non-zero components of the energy momentum tensor are

$$T_{00} = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) = \rho_\varphi;\ T_{ij} = \left(\frac{1}{2}\dot{\varphi}^2 - V(\varphi)\right)\delta_{ij} = p_\varphi\delta_{ij}.$$

In the slow-roll regime $\dot{\varphi}^2 \ll V(\varphi)$ and, consequently, $p_\varphi \approx -\rho_\varphi$. Therefore the energy-momentum tensor in the slow-roll regime approximately coincides with the vacuum one with $p = -\rho$. Using (18) and taking into account the fact that the inflation scenery with exponential growth of the scale factor leads to the flat space case, and therefore $\Omega = 1$, one obtains that $q = -1$ during the period of the inflation expansion.

Therefore the scalar field provides rather natural way to obtain the accelerated expansion of Universe at least on early stages of its evolution. As the field intensity decrease in the slow-roll regime, viscosity falls down and the Universe comes out the inflation regime with exponential growth of the scale factor.

We remark that the scalar field can provide decelerated expansion of Universe as well as the accelerated one. Near the minimum of the inflation potential the inflation conditions definitely violate and the Universe leaves the inflation regimes. The scalar field start to oscillate near the minimum. Assuming that period of the oscillations is much less than the cosmological time scale, one can neglect the expansion term in the equation (31), and it easy to come up with effective state equation near the minimum of inflation potential. Let us rewrite the scalar field equation(31) in the form

$$\frac{d}{dt}(\varphi\dot{\varphi}) - \dot{\varphi}^2 + \varphi V'_\varphi = 0.$$

After averaging over the oscillation period the first term turns to zero and then

$$\left\langle \dot{\varphi}^2 \right\rangle \simeq \left\langle \varphi V'_\varphi \right\rangle.$$

The effective (averaged) state equation now reads

$$w \equiv \frac{p}{\rho} \simeq \frac{\left\langle \varphi V'_\varphi \right\rangle - \langle 2V \rangle}{\left\langle \varphi V'_\varphi \right\rangle + \langle 2V \rangle}.$$

For the case of quadratic potential $V \propto \varphi^2$ one obtains $w \simeq 0$, which corresponds to the state equation of non-relativistic matter.

We note also that the scalar field models are very wide-spread in modern cosmology, allowing to describe not only accelerated expansion, but much more complicated dynamics of Universe. Besides, most of the scalar field models are well motivated in the particle physics and concurring theories.

IV. EVIDENCE FOR THE ACCELERATED EXPANSION OF UNIVERSE

The Hubble law tells nothing about the magnitude, sign or very possibility of non-uniform expansion of Universe. The approximation where it works is insensible to acceleration. Investigation of the non-linear effects requires high red-shift data. If the observation detect deviations from the linear law, then the magnitude and sign of the deviation allow to judge about the sign of the cosmological acceleration. If the detected deviation lies towards larger distances at fixed redshift then the acceleration is positive. The distance is estimated by brightness of the source in assumption that the considered kind of sources represent the standard candles — an ensemble of object with practically equal luminosity. Therefore the observed brightness of the objects depends only on the distance to observer. The supernova outbursts of type Ia (exploding white dwarfs) present an example of such kind of objects. As the white dwarfs have very low mass dispersion, their luminosity is practically the same. An additional advantage is the huge power of $\left(\sim 10^{36} W\right)$, released in explosion. Thanks to that, they can be detected on distances compatible to size of the observable Universe.

Given the emitted light intensity L, or internal luminosity of the object, and having measured the light intensity F, which have reached us, or observed flow, one can calculate the distance to the object. The quantity defined this way is called the luminosity distance d_L

$$d_L^2 = \frac{L}{4\pi F}. \tag{32}$$

In order to determine the acceleration of the Universe expansion one needs to express the luminosity distance in terms of the redshift for the registered radiation. Let E be internal (absolute) luminosity of some source. A terrestrial observer registers the photon flow F. Increasing of the photon wavelength, and therefore decrease of its energy, in the expanding Universe during its pass from the source to the observer results in effective (apparent) luminosity of the source $L = E/a(t)$. The conservation law for the energy emitted on time interval dt and absorbed on the interval dt_0, implies

$$F 4\pi r^2 dt_0 = E dt = L a(t) dt, \tag{33}$$

where r is the comoving distance between the source and the observer at time moment t_0, which coincides with the physical distance in normalization of the form $a(t_0) = 1$. Since the comoving distance between source and observer does not change, the conformal time interval $d\eta = dt/a$ between two light pulses is the same at the point of emission and at the point observation

$$\frac{dt}{a(t)} = \frac{dt_0}{a_0}.$$

Therefore from (33) follows that

$$F = \frac{La^2(t)}{4\pi r^2}. \tag{34}$$

Comparing this expression with the definition of the luminosity distance (32), one finds

$$d_L = \frac{r}{a(t)} = (1+z)r. \tag{35}$$

The obtained result is physically clear. In the expanding Universe the registered flow decreases by factor $(1+z)^2$: first, due to increase of the photon wave length $(1+z)$ times and, second, due to increase of the time interval for arrival of the fixed energy portion also $(1+z)$ times.

Let us now determine the comoving distance to the currently observed light source as the function of its redshift. The equation of motion for photon is $ds^2 = 0$. Let us consider a radial trajectory with the observer at the origin of the coordinates. In the case of spatially flat metrics one has

$$ds^2 = a^2(t)\left(d\eta^2 - dr^2\right) = 0. \tag{36}$$

Accounting that

$$d\eta = \frac{d\eta}{dt}\frac{dt}{da}\frac{da}{dz}dz = -\frac{dz}{H(z)}, \tag{37}$$

one finds

$$r(z) = \int_0^z \frac{dz'}{H(z')}. \tag{38}$$

Therefore for spatially flat Universe

$$d_L = (1+z)\int_0^z \frac{dz'}{H(z')}. \tag{39}$$

In general case,

$$d_L(z) = c(1+z)(1-\Omega_0)^{-1/2}H_0^{-1}S\left[(1-\Omega_0)^{1/2}H_0\int_0^z \frac{dz'}{H(z')}\right], \tag{40}$$

where

$$S(x) = \begin{cases} \sin x, & \Omega_0 > 1; \\ x, & \Omega_0 = 1; \\ \sinh x, & \Omega_0 < 1. \end{cases} \tag{41}$$

The quantities H_0, $\Omega_0 \equiv \frac{\rho_0}{\rho_{cr}}$, $\left(\rho_{cr} \equiv \frac{3H_0^2}{8\pi G}\right)$ refer to the present moment of time. For the multi-component flat case

$$d_L = \frac{1+z}{H_0}\int_0^z \frac{dz'}{\sqrt{\sum\limits_i \Omega_{0i}(1+z')^{3(1+w_i)}}}. \tag{42}$$

The relation (40). can be rewritten in terms of the deceleration parameter. For the flat case it is

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')} = (1+z)H_0^{-1}\int_0^z du \exp\left[-\int_0^u [1+q(v)d\ln(1+v)]\right]. \tag{43}$$

Let us use normal definition for the absolute value μ of the distance to a standard candle (say, SNe1a):

$$\mu(z) \equiv [m_B(z) - M_B] = 5\log\left(d_L/Mpc\right) + 25. \tag{44}$$

Using (43), one can link that quantity to the acceleration history $q(z)$ via

$$\mu(z) = 25 + 5\log\left[\frac{1+z}{H_0 Mpc}\right]\int_0^z du \exp\left(-\int_0^u [1+q(u)]\, d\ln u\right). \tag{45}$$

Here M_B and m_B are absolute and apparent stellar magnitude of the source respectively. The latter expression represents a fundamental relation, linking the deceleration parameter history with the $SNe1a$ measurements.

We remark that the relation (43) is based solely on the FRW-metrics. It means that the acceleration-deceleration dilemma can be resolved without any assumption about applicability of general relativity. However, supernova observations cannot help to determine current deceleration parameter immediately. In order to interpret the data, we must know $H(z)$ or $q(z)$, which in turns requires to specify the dynamical equations and material composition of the Universe.

It is useful to cite the expression for the luminosity distance with accuracy up to z^2 terms:

$$d_L = \frac{z}{H_0}\left[1 + \left(\frac{1-q_0}{2}\right) z + \mathrm{O}(z^2)\right], \tag{46}$$

where in the flat case $q_0 = \frac{1}{2}\sum\limits_i \Omega_i(1+3w_i)$.

It follows from (46) that for small z the luminosity distance is proportional to the redshift, with the proportionality coefficient equal to inverse value of the Hubble constant. For more distant cosmological objects the higher order corrections for the luminosity distance depend on the current value of the deceleration parameter q_0, ore, in other words, on quantity and type of components, filling the Universe.

The expression for luminosity distance in the next order correction over the redshift reads

$$\begin{aligned} d_L(z) &= \frac{cz}{H_0}[1 + \frac{1}{2}\left(1-q_0\right) z - \frac{1}{6}\left(1 - q_0 - 3q_0^2 + j_0\right) z^2 + \\ &+ \frac{1}{24}\left(2 - 2q_0 - 15q_0^2 - 15q_0^3 + 5j_0 + 10j_0q_0 + s_0\right) z^3 + \mathrm{O}(z^4)]. \end{aligned} \tag{47}$$

Let us briefly discuss the methodology for verification of the cosmological models using the type $SNe1a$ supernova outbursts. Let us consider more closely [3, 4] two supernovae 1992P at low redshift $z = 0.026$ with $m = 16.08$ and 1997ap at high redshift $z = 0.83$ with $m = 24.32$. As indicated earlier, $d_L(z) \simeq z/H_0$ for $z \ll 1$. Using (44) we find $M = -19.09$. Then the luminosity distance of 1997ap is obtained by substituting $m = 24.32$ and $M = -19.09$ for (44)

$$H_0 d_L \simeq 1.16 \quad for \quad z = 0.83. \tag{48}$$

From the other hand, for the case of Universe filled by non-relativistic matter, using (42), one finds

$$H_0 d_L \simeq 0.95.$$

The latter result apparently contradicts the observations (48).

Therefore, registration of outbursts for any kind of light sources with the same internal luminosity, i.e. standard candles, allows to determine the velocity of Universe expansion in different moments of its evolution. Comparing the obtained results with the predictions of different theoretical models, one can select the most adequate ones. Despite of the conceptual simplicity, the problem have been continuously facing many obstacles. Let us list just a few of them. The supernova outbursts are rare and random. To collect sufficient statistics one has to cover rather large part of the sky. The outbursts takes place for a limited time, therefore it is important to detect a supernova as soon as possible in order to follow the dynamics of its brightness variation. And of course the main problem is still connected to disputable applicability of the type Ia supernova for a standard candle. In the early ninetieth the United States created two groups for detection and analysis of the type $SNeIa$ supernova outbursts: SuperNova Cosmology Project and High-Z SuperNova Search. It were their results [3, 4] which in the years 1998 and 1999 gave base for conclusion on accelerated expansion of Universe, so dramatically affected not only modern cosmology, but all physics as a whole. During the passed decade the results [3, 4] were multiple times reproduced with ever improving statistics. The main conclusion was always reconfirmed: comparable recently (at $z \sim 0.5$) our Universe passed through the transition from decelerated expansion to accelerated one. Let us present in more details the analysis carried out by Riess [44], where he used the so-called golden set of $SNe1a$. The set contained 157 well studied $SNe1a$ supernova with redshifts ranged

in $0.1 < z < 1.76$. The analysis was based on the relation (43) which is exact expression for the luminosity distance in a geometrically flat Universe. In the case of linear two-parameter decomposition

$$q(z) = q_0 + q_1 z, \tag{49}$$

the integral in (43) can be exactly evaluated and the expression for the luminosity distance takes the form [45]

$$d_L(z) = \frac{1+z}{H_0} e^{q_1} q_1^{q_0 - q_1} \left[\gamma\left(q_1 - q_0, (1+z)q_1\right) - \gamma\left(q_1 - q_0, q_1\right)\right], \tag{50}$$

where q_0, q_1 are values of $q(z)$, $\frac{dq(z)}{dz}$ at $z = 0$, γ is the reduced gamma-function. Using (50) one can obtain information about q_0, q_1 and, therefore, about global behavior of $q(z)$. The dynamical "phase transition" occurs at $q\left(z_t\right) = 0$ or equivalently at $z_t = -q_0/q_1$.

The second widely used parameterization is

$$q(z) = q_0 + q_1 \frac{z}{1+z}. \tag{51}$$

Its advantage is in the fact that it is well-behaved at large z, while the linear approximation suffers of divergences. In such parameterization one has

$$d_L(z) = \frac{1}{(1+z)H_0} e^{q_1} q_1^{-(q_0+q_1)} \left[\gamma\left(q_1 + q_0, q_1\right) - \gamma\left(q_1 + q_0, q_1/(1+z)\right)\right]. \tag{52}$$

Now the parameter q_1 determines the correction to q_0 in the distant past: $q(z) = q_0 + q_1$ at $z \gg 0$. The likelihood for the parameters q_0 and q_1 can be determined from χ^2 statistic,

$$\chi^2\left(H_0, q_0, q_1\right) = \sum_i \frac{\left(\mu_{p,i}\left(z_i; H_0, q_0, q_1\right) - \mu_{0,i}\right)^2}{\sigma^2_{\mu_{0,i}} + \sigma_v^2}, \tag{53}$$

where $\sigma_{\mu_{0,i}}$ is the uncertainty in the individual distance modules, σ_v is the dispersion in SNe redshifts due to peculiar velocities. The obtained results give evidence in favor of the Universe with recent acceleration ($q_0 < 0$) and previous deceleration ($q_1 > 0$) with 99.2% and 99.8% likelihood. In the case of linear decomposition of the parameter q the transition from decelerated expansion in the past to current accelerated expansion took place at the redshift $z_t = 0.46 \pm 0.13$. Unfortunately one should not take this result too serious: the linear approximation always leads to the transition, provided the two parameters have opposite signs.

The consequent and statistically more reliable analysis qualitatively supported the above cited results. Of course, the quantitative results depend on the used set of supernova, but the main result remains the same: we live in accelerated expanding Universe, which passed from decelerated expansion to accelerated one in the recent past.

Nowadays the method based on the supernova outbursts observations is undoubtedly leading. But it turned out to have competitors. A promising and completely independent alternative (and no way a surrogate) is the observation of angular diameter distances $D_A(z)$ for a given set of distant objects. The combination of the Sunyaev-Zeldovich effect with measurements of surface brightness in X-ray range provides the method for determination of angular diameter distances for galactic clusters [47]. The Sunyaev-Zeldovich stands for small perturbation of CMB spectrum produced by inverse Compton scattering of relict photons passed through clouds of hot electrons. Observing the temperature decrement of the CMB spectrum towards galaxy clusters together with the X-rays observations, it is possible to break the degeneracy between concentration and temperature thereby obtaining $D_A(z)$. Therefore, such distances are fully independent of the one given by the luminosity distance.

For the case of spatially flat Universe described by FRW-metric, the angular diameter distance is

$$d_A(z) = \frac{1}{(1+z)H_0} \int_0^z \frac{du}{H\left(u\right)} = \frac{1}{(1+z)H_0} \int_0^z \exp\left[-\int_0^u \left[(1 + q(u')\right] d\ln(1+u'\right] du, \tag{54}$$

[46] considered the 38 measurements of angular diameter distances from galaxy clusters as obtained through SZE/X-ray method by Bonamente and coworkers [47] Lima et al used a maximum likelihood determined by a χ^2 statistics

$$\chi^2\left(z, p\right) = \sum_i \frac{\left(D_A\left(z_i, p\right) - D_{A_{0,i}}\right)}{\sigma^2_{D_{A0i}} + \sigma^2_{stat}}, \tag{55}$$

where $D_{A_{0,i}}$ is the observational angular diameter distance, $\sigma_{D_{A0i}}$is the uncertainty in the individual distance, σ_{stat} is the contribution of the statistical errors added in quadrature and the complete set of parameters is given by $p \equiv (H_0, q_0, q_1)$. For the sake consistency, the Hubble parameter H_0 has been fixed by its best it value $H_0^* = 80km/\sec\cdot Mpc^{-1}$. In the case of linear parameterization the results are the following: best its to the free parameters are $q_0 = -1.35,\ q_1 = 4.2,\ z_t = 0.32$. Such results favor a Universe with recent acceleration ($q_0 < 0$) and a previous decelerating stage $(dq/dz > 0)$. In the case of the parameterization

$$q(z) = q_0 + q_1 \frac{z}{1+z},$$

one gets $q_0 = -1.43,\ q_1 = 6.18,\ z_t = 0.3$. In both cases the results well agree with the ones obtained using the $SNe1a$ data.

It was recently shown that the so-called luminous red galaxies (LRG's) provide another possibility for direct measurements of the expansion velocity [48, 49, 55]. Idea of the method is to reconstruct the Hubble parameter from the time derivative of the redshift

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt}. \tag{56}$$

The derivative can be found in measurements of the "age difference " between two passively evolving galaxies at different but close redshifts. The method was realized for $0.1 < z < 1.75$. We stress that the range includes the interesting for us transition region, where $z \sim 0.5$. The results of analysis for available data [56] agree with the ones based on the supernova and angular diameter. In nearest future a set of some 2000 passively evolving galaxies is going to be measured in the range $0 < z < 1.5$. Those observations will give about 1000 values of $H(z)$ with 15% accuracy, provided the age of galaxies is determined with 10% precision.

Concluding the present section we present a novel and promising direction to research the history of cosmological expansion of Universe. We remind that astrophysicists initially used cepheids — the variable stars with intensity proportional to period of brightness variation — as the standard candles. A typical example of cepheids is the Polar Star, the brightest and closest to Earth variable star with period of 3.97 days. The cepheids are perfect standard candles for galactic distances. They allowed to determine size of our Galaxy and distance to our closest neighbor — the Andromeda galaxy. Research of the Universe dynamics involves principally larger scales, and thus it requires to use essentially more powerful standard candles. Recall, that the cosmological principle, which postulates homogeneity and isotropy of the Universe, and which all our above used equations of Universe dynamics are based on, is valid on scales of 100 Mpc and larger. Using considerably more powerful radiation sources as the standard candles, namely type Ia supernovae, already allowed us to advance remarkably deeper to the history of Universe. However the possibilities of the new standard candles are also limited. Up to present time the type Ia supernovae were observed only up to the redshift $z < 2$, though more reliable reconstruction of the cosmological expansion history requires even higher redshifts and therefore more powerful standard candles. It turns out that objects with required properties are already available! It is the so-called gamma-ray bursts(GRB) — giant energy emissions of explosive type with duration from tree to hundred second, observed in the hardest part of the electromagnetic spectrum. The typical energy, emitted by the gamma-ray burst is $\sim 10^{54}\, erg$ which is one order of magnitude higher than at a supernova outburst. It is already compatible to the Sun rest mass! The events producing the gamma-ray bursts are so powerful that they can be observed even by naked eye, though they occur on distances of billions light years from the Earth! The energy yields in form of a collimated flow, called a jet. Occurrence of the jets means that we can see only small number of all bursts that happen in Universe. Distribution of GRB durations has pronounced bimodal character.

Origin of short GRB is probably linked to coalescence of two neutron stars, or a neutron star and a black hole. The events with longer duration are presumably due to creation of a black hole in collapse of massive stellar nucleus (¿25 Solar mass) with high angular momentum — it is the so-called collapsar model. The possibility to use the GRB as the standard candles is based on occurrence of the so-called "Amati relation", which relates the peak frequency of the burst with its total energy. This relation is a direct analog of the period-luminosity relation for cepheids. Strong dispersion (see Fig.1) yet limits applications of the GRB as the standard candles, but the possibility to advance to the region of considerably higher redshifts makes this direction potentially very attractive.

V. SCALE FACTOR DYNAMICS IN SCM

Now let us consider evolution of the deceleration parameter in Standard Cosmological model (SCM). Recall that the Big Bang model included only two substances – matter and radiation – which could give only decelerated expansion

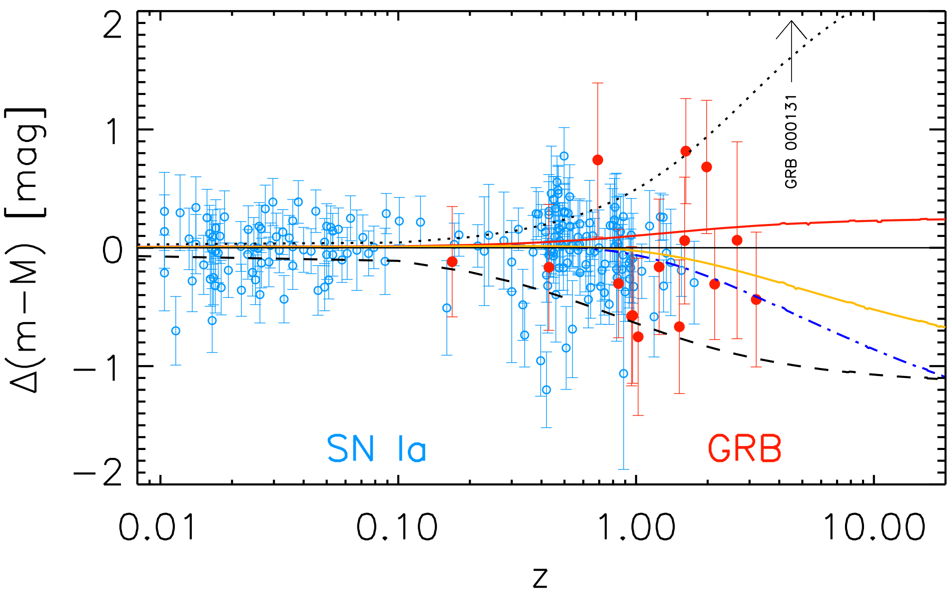

FIG. 1: 15 GRB with measured redshifts and collimation angle(compared to Ia supernova data in SCM)

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of the Universe. According to SCM, the Universe is presently dominated by the dark energy — a component with negative pressure. It is the component responsible for the observed accelerated expansion of Universe. Let us determine the redshift and time moment of the transition to the accelerated expansion, i.e. find the inflection point on the curve (see Fig. 2), describing the time dependence of the scale factor The second Friedmann equation in the SCM (14) can be transformed to the following

$$\frac{\ddot{a}}{a} = \frac{1}{2}H_0^2\left[2\Omega_{\Lambda 0} - \Omega_{m0}(1+z)^3\right], \tag{57}$$

where $\Omega_{\Lambda 0}$ and Ω_{m0} are present values of the relative density for dark energy in form of cosmological constant and for matter respectively. Now it follows for the redshift at the point of transition from decelerated expansion to accelerated one

$$z^* = \left(\frac{2\Omega_{\Lambda 0}}{\Omega_{m0}}\right)^{1/3} - 1. \tag{58}$$

With the SCM parameters $\Omega_{\Lambda 0} \simeq 0.73,\ \Omega_{m0} \simeq 0.27$ one gets $z^* \simeq 0.745$. Note that the result (58) can be obtained using the fact that for a multi-component Universe with state equations $p_i = w_i\rho_i$ respectively, the deceleration parameter (18) equals (recall that $\Omega = \sum\limits_i \Omega_i = 1$ in the flat case)

$$q = \frac{1}{2} - \frac{3}{2}\frac{\Omega_{\Lambda 0}}{(1+z)^3\Omega_{m0} + \Omega_{\Lambda 0}}. \tag{59}$$

The condition $q = 0$ allows to reproduce the relation (58). Let us discuss its asymptotes. For early Universe ($z \to \infty$), filled by components with positive pressure , $q\,(z \to \infty) = \frac{1}{2}$, i.e. as it was expected the expansion is decelerated, while in the distant future with domination of the cosmological constant the accelerated expansion is observed: $q\,(z \to -1) = -1$. The latter result is a trivial consequence of the exponential expansion $a \propto e^{Ht}$ in the case of Universe dominated by dark energy in the form of cosmological constant.

We remark that the dependence $q(t)$ can be immediately obtained from the definition $q = -\ddot{a}/aH^2$, using the SCM solutions for the scale factor:

$$\begin{aligned} a(t) &= A^{1/3}\sinh^{2/3}(t/t_\Lambda)\,; \\ A &\equiv \frac{\Omega_{m0}}{\Omega_{\Lambda 0}}, \quad t_\Lambda \equiv \frac{2}{3}H_0^{-1}\Omega_{\Lambda 0}^{-1/2}. \end{aligned} \tag{60}$$

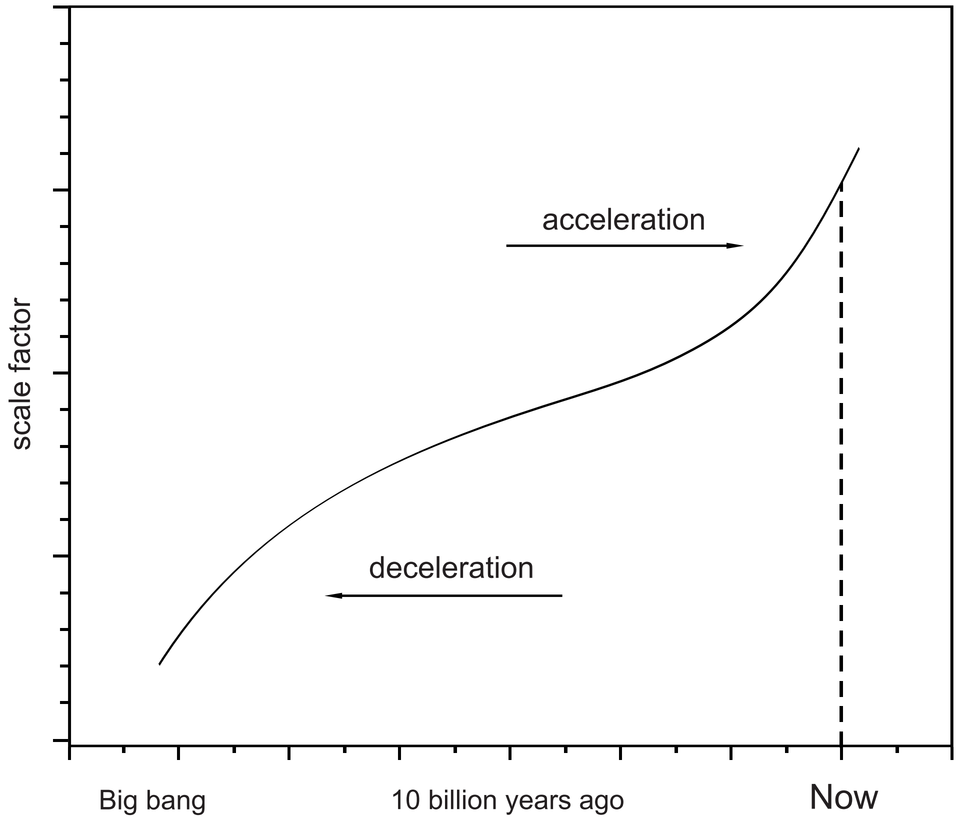

FIG. 2: Time dependence of the scale factor in SCM.

It results in the following

$$q(t)=\frac{1}{2}\left[1-3\tanh^2\left(\frac{t}{t_\Lambda}\right)\right], \tag{61}$$

The dependence $q(t)$ is presented on Fig.3. Note that the time asymptotes at $t\to 0$ and $t\to\infty$ correspond to the above obtained ones at $z\to\infty$ and $z\to -1$.

Let us now determine the time of the transition to the accelerated expansion. Inserting the relation (60), one obtains

$$t\,(a)=\frac{2}{3}\Omega_{\Lambda 0}^{-1/2}H_0^{-1}arsh\left[\left(\frac{\Omega_{\Lambda 0}}{\Omega_{m0}}\right)^{1/2}a^{3/2}\right]. \tag{62}$$

Transforming from the redshift to the scale factor $a^*=(1+z^*)^{-1}=\left(\frac{\Omega_{m0}}{2\Omega_{\Lambda 0}}\right)^{1/3}$, one gets

$$t^*\equiv(a^*)=\frac{2}{3}\Omega_{\Lambda 0}^{-1/2}H_0^{-1}\sinh^{-1}(1/2)\simeq 5.25\,Gyr. \tag{63}$$

Because of physical importance of the obtained result we present another one, probably the simplest, interpretation of it. If the quantity $aH=\dot a$ growth, then $\ddot a>0$, which corresponds to accelerated expansion of Universe. According to the first Friedmann equation,

$$\frac{aH}{H_0}=\sqrt{\frac{a^3\Omega_{\Lambda 0}+\Omega_{m0}}{a}}\simeq\sqrt{\frac{0.73a^3+0.27}{a}}.$$

It is easy to show that the function under radical starts to grow at $a^*\simeq 0.573$, which corresponds to $z^*=0.745$. It is interesting to note that the transition to accelerated expansion of Universe ($z\simeq 0.75$) occurred remarkably earlier than the dark energy started to dominate ($z\simeq 0.4$).

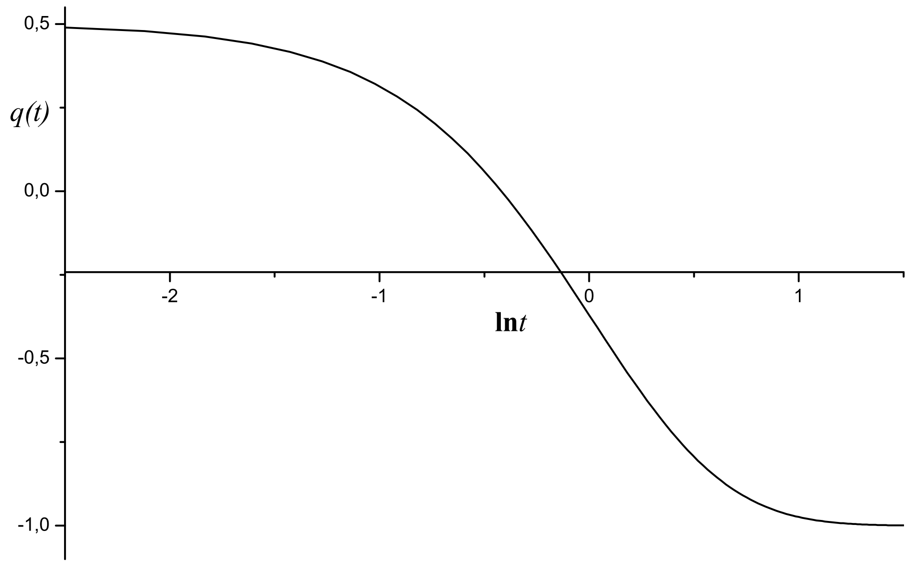

FIG. 3: Time dependence of the deceleration parameter $q(t)$ in SCM.

In the SCM the dark energy exist in form of cosmological constant with the state equation $p_\Lambda = -\rho_\Lambda$, i.e. the state parameter equals $w_\Lambda = -1$. A natural question rises: what is the limiting value of that parameter, which still provides accelerated expansion of Universe in the present time? As we have seen above, the condition for accelerated expansion is the following $\sum_i (\rho_i + 3p_i) < 0$. In SCM it is transformed to the following

$$w_{DE} < -\frac{1}{3}\Omega_{DE}^{-1}; \quad w_{DE} < -0.46.$$

Of course, a substance with such state equation is not the cosmological constant, and it can be realized, for instance, with the help of scaler fields (see next section).

Now let us estimate absolute magnitude of the cosmological acceleration using the SCM parameters. Taking time derivative of the Hubble law, one gets

$$\dot{V} = \left(\dot{H} + H^2\right) R. \tag{64}$$

The time derivative of the Hubble parameter is

$$\dot{H} = \frac{\ddot{a}a - \dot{a}^2}{a^2} = \frac{\ddot{a}}{a} - H^2.$$

Therefore

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_m - 2\rho_\Lambda\right) = \frac{8\pi G}{3}\left(\rho_\Lambda - \frac{1}{2}\rho_m\right) = H^2\left(\Omega_\Lambda - \frac{1}{2}\Omega_m\right).$$

And finally we get the analog of the Hubble law for the acceleration $\dot{V}$

$$\dot{V} = \tilde{H}R; \quad \tilde{H} = H^2\left(\Omega_\Lambda - \frac{1}{2}\Omega_m\right). \tag{65}$$

In the present time ($\Omega_m = \Omega_{m0},\ \Omega_\Lambda = \Omega_{\Lambda 0}$), and on the distance, for example, $R = 1$ Mpc one gets

$$\dot{V} \simeq 10^{-11} cm \sec^{-2}. \tag{66}$$

A possible way to observe such effect is based on the fact that the redshift for any cosmological object slowly changes due to the acceleration (or deceleration) of the Universe expansion. Let us estimate the magnitude of that effect. By definition

$$z = \frac{a\,(t_0)}{a(t)} - 1,$$

where t is the time of emission and t_0 is the detection time, and it follows that

$$\frac{dz}{dt_0} = \frac{\dot{a}(t_0)a(t) - a(t_0)\dot{a}(t)\frac{dt}{dt_0}}{a^2(t)}. \tag{67}$$

Taking into account that $\frac{dt}{dt_0} = \frac{a(t)}{a(t_0)} = \frac{1}{(1+z)}$, one gets

$$\dot{z} = \frac{\dot{a}(t_0)}{a(t)} - \frac{a(t_0)\dot{a}(t)}{a^2(t)}\frac{1}{1+z}. \tag{68}$$

Thus, the rate of redshift variation for the light emitted at time t and registered at the present time t_0, is determined by the relation

$$\dot{z} \equiv \frac{dz}{dt_0} = H_0(1+z) - H(t).$$

In the SCM

$$H = H_0(\Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0})^{1/2}.$$

Therefore for variation of the redshift on the time interval Δt one obtains

$$\Delta z = \dot{z}\Delta t = H_0\left[1 + z - (\Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0})^{1/2}\right]. \tag{69}$$

Note that for the two limiting cases $\Omega_{\Lambda 0} = 1$, $\Omega_{m0} = 0$ (accelerated expansion) and $\Omega_{\Lambda 0} = 0$, $\Omega_{m0} = 1$ (decelerated expansion), as was expected Δz has opposite sign. For the SCM parameters the redshift variation Δz and the velocity increment ΔV for a source with the redshift $z = 4$ and duration of observation $\Delta t_0 = 10$ equal respectively

$$\Delta z \approx 10^{-9}, \quad \Delta V = c\frac{\Delta z}{1+z} \approx 6\,cm/\sec.$$

The result discourages by its smallness. However, taking into account fast progress in precision of observable cosmology, we should not despair. Let us give an example. Today the list of exoplanets – planets out the Solar system – counts more than 300 items. The most successful method to detect the exoplanets is to measure radial velocity of the stars. A star accompanied by an exoplanet experiences velocity oscillations "to us and back from us", which can be measured observing the Doppler shift of the star spectrum. For the first look it is impossible: affected by the Earth, velocity of the sun annually oscillates with amplitude of centimeters per second. Even effect of Jupiter produces variations of only meters per second, while thermal broadening of the spectral lines for the star corresponds to dispersion of velocity of order of thousands kilometers per second. It means that even in the case of Jupiter one have to measure shift of the spectral lines on a thousandth of its width. It seems incredible, but the task was successfully worked out.

VI. DYNAMICAL FORMS OF DARK ENERGY AND EVOLUTION OF UNIVERSE

The cosmological constant represents one of many possible realizations of the hypothetic substance called the dark energy, introduced for explanation of the accelerated expansion of Universe. As we have seen above, such substance have to have the parameter w in the state equation $p = w\rho$, which satisfies the condition $w < -1/3$ (in absence of other components). Unfortunately the nature of dark energy is absolutely unknown, which produces huge number of hypotheses and candidates for the role of fundamental contributor to the energy budget of the Universe. We told many times about impressive progress of observational cosmology in the last decade. However we still cannot answer the question about time evolution of the state parameter w. If it changes with time, we must seek an alternative to the cosmological constant. For quite a short time a plenty of such alternative (with respect to $w = -1$) possibilities were investigated. The scalar fields, formed the post-inflation Universe, are considered to be one of the principal candidates

for the role of dark energy. The most popularity acquired the version of the scalar field φ with appropriately chosen potential $V(\varphi)$. In such models, unlike the cosmological constant one, the scalar field is truly dynamical variable, and dark energy density depends on time. The models differ by the choice of the scalar field Lagrangian. Let us start from probably the simplest dark energy model of such type, named the quintessence. We define quintessence as the scalar field φ in the potential $V(\varphi)$, minimally coupled to gravity, i.e. experiencing effect of the space-time curvature. Besides that we chose canonical form for the kinetic energy term. The action for such field takes the form

$$S = \int d^4x\sqrt{-g}L = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\frac{\partial\varphi}{\partial x_\mu}\frac{\partial\varphi}{\partial x_\nu} - V(\varphi)\right], \tag{70}$$

where $g \equiv \det g_{\mu\nu}$. Equation of motion for the scalar field is obtained by variation of the action with respect to the field,

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\frac{\partial\varphi}{\partial x_\nu}\right) = -\frac{dV}{d\varphi}. \tag{71}$$

In the case of flat Friedmannian Universe, namely in FRW-metric (1) one obtain for homogeneous field $\varphi(t)$ the following

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0, \tag{72}$$

where $V'(\varphi) \equiv \frac{dV}{d\varphi}$. This relation is sometimes called the Klein-Gordon equation.

The energy-momentum tensor for the scalar field can be found by variation of (70) with respect to metrics $g^{\mu\nu}$, resulting in the following

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \frac{\partial\varphi}{\partial x_\mu}\frac{\partial\varphi}{\partial x_\nu} - g_{\mu\nu}L. \tag{73}$$

In the case of homogeneous field$\varphi(t)$ in locally Lorentzian reference frame, where metrics$g_{\mu\nu}$ can be replaced by the Minkowski one, we obtain the density and pressure of the scalar field in the following form

$$\rho_\varphi = T_{00} = \frac{1}{2}\dot{\varphi}^2 + V(\varphi); \quad p_\varphi = T_{ii} = \frac{1}{2}\dot{\varphi}^2 - V(\varphi). \tag{74}$$

The Friedmann equations for the flat Universe, filled by the scalar field, take the form

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right]; \tag{75a}$$

$$\dot{H} = -4\pi G\dot{\varphi}^2. \tag{75b}$$

The system (75a – 75b) should be complemented by the Klein-Gordon equation (72) for the scalar field. Note, that the latter can be obtained from the conservation equation for the scalar field

$$\dot{\rho}_\varphi + 3H(\rho_\varphi + p_\varphi) = 0. \tag{76}$$

by substitution of the expressions (74) for energy density and pressure.

Using (74), one get the state equation for the scalar field

$$w_\varphi = \frac{p_\varphi}{\rho_\varphi} = \frac{\dot{\varphi}^2 - 2V}{\dot{\varphi}^2 + 2V}. \tag{77}$$

As one can see, the parameter of the state equation for the scalar field w_φ varies in the range

$$-1 \le w_\varphi \le 1. \tag{78}$$

The state equation for the scalar field is conveniently transformed into the following

$$w(x) = \frac{x-1}{x+1}, \quad x \equiv \frac{\frac{1}{2}\dot{\varphi}^2}{V(\varphi)}. \tag{79}$$

The function $w(x)$ increases monotonously from the minimum value $w_{\min} = -1$ at $x = 0$ up to the maximum asymptotic value $w_{\max} = 1$ at $x \to \infty$, corresponding to $V = 0$. In the slow-roll limit, $x \ll 1$ ($\dot{\varphi}^2 \ll V(\varphi)$) the scalar field behaves as the cosmological constant, $w_\varphi = -1$. It is easy to see that in such case $\rho_\varphi = const$. The opposite limit $x \gg 1$ ($\dot{\varphi} \gg V(\varphi)$) correspond to the hard matter $w_\varphi = 1$. In that case the scalar field energy density evolves as $\rho_\varphi \propto a^{-6}$. The intermediate case $x \sim 1$, $p \sim 0$ corresponds to non-relativistic matter.

Transforming (76) into the integral form

$$\rho = \rho_0 \exp\left[-3\int (1 + w_\varphi)\frac{da}{a}\right]. \tag{80}$$

one finds that in the general case the scalar field energy density behaves as

$$\rho_\phi \propto a^{-m}, \quad 0 < m < 6. \tag{81}$$

The value $w_\varphi = -1/3$ represents boundary between the regimes of decelerated and accelerated Universe expansions. Thus, the case of accelerated expansion takes place for $0 \le m < 2$. A natural question arises: what are the potentials for scalar fields that can provide the accelerated expansion of Universe? The question can be posed another way: what potentials can produce the quintessence applicable for the role of dark energy? Consider simplified version of the problem. Let us find the scalar field potential resulting in power law for the scale factor growth:

$$a(t) \propto t^p. \tag{82}$$

For accelerated expansion the condition $p > 1$ must be satisfied. Recall that $p = 2/3$ for the case of non-relativistic matter dominated Universe, and $p = 1/2$ for radiation-dominated one, so in both cases the expansion will be decelerated. Making use of the Friedmann equations, one can express the potential $V(\varphi)$ and $\dot{\varphi}$ in terms of H and $\dot{H}$. It allows to write down the system of equations, which describe the parametric dependence V on φ :

$$V = \frac{3H^2}{8\pi G}\left(1 + \frac{\dot{H}}{3H^2}\right); \tag{83a}$$

$$\varphi = \int dt \left(-\frac{\dot{H}}{4\pi G}\right)^{1/2}. \tag{83b}$$

Excluding the time variable with the help of the relation $\frac{\varphi}{m_{Pl}} = \sqrt{\frac{p}{4\pi}} \ln t$, for the case of the power law (82) one finds

$$V(\varphi) = V_0 \exp\left(-\sqrt{\frac{16\pi}{p}}\frac{\varphi}{m_{Pl}}\right). \tag{84}$$

The obtained result implies that the scalar field in the potential (84) under condition $p > 1$ can be treated as the dark energy, i.e. it provides accelerated expansion of Universe. As we can see above, in the quintessence model the required dynamical behavior is achieved by the choice of the scalar field potential. Another dark energy model, realized by the scalar field with modified kinetic term, is called the k-essence [80, 81].

Let us define the quantity $X \equiv \frac{1}{2} g^{\mu\nu} \frac{\partial\varphi}{\partial x_\mu}\frac{\partial\varphi}{\partial x_\nu}$ and consider the action for the scalar field of the form

$$S = \int d^4x \sqrt{-g} L(\varphi, X). \tag{85}$$

where L is generally speaking an arbitrary function of the variables φ and X. The traditional scalar field action corresponds to the case

$$L(\varphi, X) = X - V(\varphi). \tag{86}$$

We restrict our consideration to the following subset of Lagrangians:

$$L(\varphi, X) = K(X) - V(\varphi), \tag{87}$$

where $K(X)$ is positive defined function of the kinetic energy X. For description of homogeneous and uniform Universe we should choose $X = \frac{1}{2}\dot{\varphi}^2$. Using the standard definition (74), one finds

$$p_\varphi = L(\varphi, X) = K(X) - V(\varphi); \tag{88a}$$

$$\rho_\varphi = 2X\frac{\partial K(X)}{\partial X} - K(X) + V(\varphi). \tag{88b}$$

Accordingly, the state equation for k-essence takes the form

$$w_\varphi = \frac{K(X) - V(\varphi)}{2X\frac{\partial K(X)}{\partial X} - K(X) + V(\varphi)}. \tag{89}$$

We demonstrate the main features of k-essence in example of the simplified model [82], where the Lagrangian has the form $L = F(X)$. Such model is called purely kinetic k-essence. In that case one has

$$\rho_\varphi = 2XF_X - F; \quad F_X \equiv \frac{\partial F}{\partial X}; \tag{90a}$$

$$p = F; \tag{90b}$$

$$w_\varphi = \frac{F}{2XF_X - F}. \tag{90c}$$

The equations of motion for the field can be obtained either from the Euler-Lagrange equation for the action (85), or by substitution of density and pressure expressions (90a and 90b) respectively into the conservation equation for the k-essence. It results in the following

$$F_X\ddot{\varphi} + F_{XX}\dot{\varphi}^2\ddot{\varphi} + 3HF_X\dot{\varphi} = 0, \tag{91}$$

or in terms of the kinetic energy X

$$(F_X + 2F_{XX}X)\,\dot{X} + 6HF_XX = 0. \tag{92}$$

The latter equation can be exactly solved

$$XF_X^2 = ka^{-6}. \tag{93}$$

where constant $k > 0$.

The solution (93) $X(a)$ has one important property: all principal characteristics of k-essence ($\rho_\varphi, p_\varphi, w_\varphi$; see (90a - 90c)) as functions of the scale factor are completely determined by the function $F(X)$ and do not depend on evolution of other energy densities. All the dependence of k-essence on other components appears only by means of $a(t)$. But this dependence is trivially $a(t) \propto \rho_{tot}$, and it is determined by the dominant component in the energy density. Solutions of such type are called the tracker solutions and its occurrence allows to approach the solution of the coincidence problem. It can be shown [83], that such property is present not only in purely kinetic k-essence, but also its general case.

Extensive set of available cosmological observations shows that the parameter w in the state equation for dark matter lies in narrow vicinity of $w = -1$. We considered above the region $-1 \le w < -1/3$. The lower bound of the region $w = -1$ corresponds to the cosmological constant, and all remaining interval can be realized by scalar fields with canonic Lagrangian. Recall that the upper bound $w = -1/3$ appears because of requirement to provide the observed accelerated expansion of Universe. But can we go beyond the bounds of that interval?

It is rather difficult question for the component of energy we know so little about. General relativity usually impose certain restrictions on possible values of the energy-momentum tensor components, which are called the energy conditions(see section III). One of the simplest restrictions of such type is the condition $\rho + p \ge 0$. Its physical motivation is to prevent the instability of vacuum. Applied to dynamics of Universe this condition requires that density of any allowed energy component should not grow as Universe expands. The cosmological constant with $\dot{\rho}_\Lambda = 0$ represents the limiting. Taking into account our ignorance about the nature of dark energy, it is reasonable to pose a question: may such a mysterious substance differ so much from already known "good" energy sources and violate the condition $\rho + p \ge 0$? Accounting the requirement of positive defined energy density, valid even for dark one (it is necessary to make the Universe flat), and negative pressure (in order to explain the accelerated Universe expansion), the above mentioned violation will result in $w < -1$. Some time ago such component, called the fantom

energy, attracted attention of physicists[86]. Action for the fantom field φ, minimally coupled to gravity, differs only by sign of the kinetic term from the canonic action for the scalar field. In that case the energy density and pressure of the fantom field are determined by the expressions

$$\rho_\varphi = T_{00} = -\frac{1}{2}\dot{\varphi}^2 + V(\varphi); \quad p_\varphi = T_{ii} = -\frac{1}{2}\dot{\varphi}^2 - V(\varphi), \tag{94}$$

and state equation takes the form

$$w_\varphi = \frac{p_\varphi}{\rho_\varphi} = \frac{\dot{\varphi}^2 + 2V(\varphi)}{\dot{\varphi}^2 - 2V(\varphi)}. \tag{95}$$

If $\dot{\varphi}^2 < 2V(\varphi)$, then $w_\varphi < -1$. As an example let us consider the case of Universe, containing only non-relativistic matter ($w = 0$) and phantom field ($w_\varphi < -1$). Densities of the two components evolve independently: $\rho_m \propto a^{-3}$ and $\rho_\varphi \propto a^{-3(1+w_\varphi)}$. If matter domination ends at the time moment t_m, then the solution for the scale factor at $t > t_m$ reads

$$a(t) = a(t_m)\left[-w_\varphi + (1 + w_\varphi)\left(\frac{t}{t_m}\right)\right]^{\frac{2}{3(1+w_\varphi)}}. \tag{96}$$

It immediately follows that for $w_\varphi < -1$ at the time moment $t_{BR} = \frac{w_\varphi}{(1+w_\varphi)}t_m$ the scale factor and whole set of the cosmological characteristics (such as scalar curvature and energy density of the fantom field) for the Universe diverge. This catastrophe was called the Big Rip, which is precede by a specific regime which is called super-acceleration. Let us explain the origin of the super-acceleration regime on a simple example. Consider differential equation

$$\frac{dx}{dt} = Ax^2. \tag{97}$$

In the case $A > 0$ the equation (93) realizes the non-linear positive feedback. Fast growth of the function $x(t)$ leads to the "Big Rip" (divergence of the function to infinity) on finite time period. Indeed, the general solution of the equation reads

$$x(t) = -\frac{1}{A(t + B)}. \tag{98}$$

where B is the integration constant. At $t = -B$ the Big Rip occurs.

It is easy to see that the model (97) represents a particular version of the Friedmann equation for $w_\varphi < -1$. Since $\rho_\varphi \propto a^{-3(1+w_\varphi)}$, then the first Friedmann equation can be presented in the form

$$\dot{a} = Aa^{-\frac{3}{2}(1+w_\varphi)+1}. \tag{99}$$

For instance, with $w_\varphi = -\frac{5}{3}$ the equation (99) exactly coincides with (98).

VII. $f(R)$-GRAVITY, BRANEWORLD COSMOLOGY AND MOND

Although SCM can explain the present accelerated expansion of Universe and well agrees with current observational data, theoretical motivation for this model can be regarded as quite poor. Consequently, there have been several attempts to propose dynamical alternatives to dark energy. We considered them in the previous section. Unfortunately, none of these attempts are problem-free. An alternative and more radical approach is based on assumption that there is no dark energy, and the acceleration is generated because of gravity weakened on very large scales due to modification of general relativity. In frames of such broad approach three main directions can be selected: $f(R)$ gravity, braneworld cosmology and modified Newtonian dynamics (MOND). Let us briefly consider those alternatives to SCM from the point of view under our interest: slowdown or speedup?

A. $f(R)-$ gravity

Theory of $f(R)$ gravity is created by direct generalization of the Einstein-Hilbert action with the replacement $R \to f(R)$. The transformed action reads

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R). \tag{100}$$

For the generalization we take the function $f(R)$ depending only on Ricci scalar R, not including other invariants, such as $R_{\mu\nu}R^{\mu\nu}$. Motivation for such choice is the following: the action with $f(R)$ is sufficiently generic in order to reflect main features of the gravity, and with all that it is simple enough to avoid technical difficulties in calculations. We remark that the function $f(R)$ must satisfy the stability conditions

$$f'(R) > 0, \quad f''(R) > 0, \tag{101}$$

were the primes denote differentiation with respect to the scalar Ricci curvature R. Complete action for $f(R)$ gravity thus reads

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}f(R) + S_m\left(g_{\mu\nu},\psi\right) \quad . \tag{102}$$

Here ψ is common symbol for all material fields. Variation with respect to metrics after some manipulations gives

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box\right)f'(R) = 8\pi G T_{\mu\nu}, \tag{103}$$

where

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}}, \tag{104}$$

and ∇_μ is the covariant derivative, associated with the Levi-Civita connection of the metric and $\Box \equiv \nabla_\mu\nabla^\mu$.

Leaving aside the complications connecting with variation procedure, let us concentrate our attention on the field equations (103). They represent partial differential equation of fourth order in metrics, as the Ricci scalar R already contains second order derivatives of the latter. For the action linear on R, the fourth-order derivatives (the two latter terms on the left hand side of (103)) turn to zero and theory reduces to the standard general relativity.

Note that the trace of (103) gives

$$f'(R)R - 2f(R) + 3\Box f'(R) = 8\pi G T, \tag{105}$$

where $T = g^{\mu\nu}T_{\mu\nu}$ links R with T differential relation, unlike algebraic one in general relativity, where $R = -8\pi G T$. This a direct hint on the fact that the field equation of the $f(R)$-theory allow much wider set of solutions compared to general relativity. To illustrate such statement we remark that the Jebsen-Birkhoff's theorem, stating that the Schwarzschild solution represents the unique spherically symmetric vacuum solution, does not hold in the $f(R)$-theory. Leaving aside the details, we note, that now $T = 0$ does not require that $R = 0$ or even is constant.

The equation (105) appears very useful for investigation of different aspects of $f(R)$-gravity, especially for stability of solutions and weak field limit. In particular it is conveniently used for analysis of the so-called maximally symmetric solutions. It is the solutions with $R = const$. For $R = const$ and $T_{\mu\nu} = 0$ the equation (105) reduces to

$$f'(R)R - 2f(R) = 0. \tag{106}$$

For a given function $f(R)$ this equation is algebraic for R. If $R = 0$ is a root of the equation, then (103) reduces in that case to $R_{\mu\nu} = 0$ and maximally symmetric solution represents the Minkovski space-time. If otherwise the root of equation (106) equals $R = const = C$, then (103) reduces to $R_{\mu\nu} = \frac{C}{4}g_{\mu\nu}$ and maximally symmetric solution corresponds to the space of de Sitter or anti-de Sitter (the cosmological constant in general relativity) depending on sign of C.

Now let us consider how dynamics of Universe are immediately described in $f(R)$ cosmology. Inserting FRW-metric into (103) and using the stress-energy tensor in the form

$$T_{\mu\nu} = \left(\rho + p\right)u_\mu u_\nu - p g_{\mu\nu}, \tag{107}$$

one obtains

$$H^2 = \frac{8\pi G}{3f'}\left[\rho + \frac{1}{2}\left(Rf' - f\right) - 3H\dot{R}f''\right]; \tag{108}$$

$$2\dot{H} + 3H^2 = -\frac{8\pi G}{f'}\left[p + \dot{R}^2 f''' + 2H\dot{R}f'' + \ddot{R}f'' + \frac{1}{2}\left(f - Rf'\right)\right]. \tag{109}$$

As was mentioned above, the main motivation for $f(R)$-gravity is in the fact that it leads to accelerated expansion of Universe without any dark energy. The easiest way to see it is to introduce the effective energy density and effective pressure

$$\rho_{eff} = \frac{Rf' - f}{2f'} - \frac{3H\dot{R}f''}{f'}; \tag{110}$$

$$p_{eff} = \frac{\dot{R}^2 f''' + 2H\dot{R}f'' + \ddot{R}f'' + \frac{1}{2}(f - Rf')}{f'}. \tag{111}$$

In the spatially flat Universe the density ρ_{eff} must be non-negative, as it follows (108) in the limit $\rho \to 0$. Then the equations (108), (109) take the form of standard Friedmann equations

$$H^2 = \frac{8\pi G}{3}\rho_{eff}; \tag{112}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_{eff} + 3p_{eff}\right). \tag{113}$$

The effective parameter w_{eff} in the state equation in that case equals

$$w_{eff} \equiv \frac{p_{eff}}{\rho_{eff}} = \frac{\dot{R}^2 f''' + 2H\dot{R}f'' + \ddot{R}f'' + \frac{1}{2}(f - Rf')}{\frac{Rf'-f}{2} - 3H\dot{R}f''}. \tag{114}$$

The denominator in (114) is definitely positive, so sign of w_{eff} is determined by the numerator. In general case of metric $f(R)$-model in order to reproduce (mimic) the equation of state for de Sitter (cosmological constant) with $w_{eff} = -1$, the following condition must be satisfied

$$\frac{f'''}{f''} = \frac{\dot{R}H - \ddot{R}}{\dot{R}^2}. \tag{115}$$

Let us consider two examples (regardless its realizability). The firs is the function of the form $f(R) \propto R^n$. We can easily calculate w_{eff} as function of n, if we assume the power law for time dependence of the scale factor $a(t) = a_0(t/t_0)^\alpha$ (arbitrary dependence $a(t)$ results in time dependence for w_{eff}). The result for $n \neq 1$ is

$$w_{eff} = -\frac{6n^2 - 7n - 1}{6n^2 - 9n + 3}, \tag{116}$$

and α in terms of n reads

$$\alpha = \frac{-2n^2 + 3n - 1}{n - 2}. \tag{117}$$

Appropriate choice of n leads to desired value for w_{eff}. For instance, $n = 2$ leads to

$$w_{eff} = -1, \quad \alpha = \infty. \tag{118}$$

This result is expected from consideration of quadratic corrections to Einstein-Hilbert action, which were used in the inflation scenery by Starobinski [88].

The second example is

$$f(R) = R - \frac{\mu^{2(n+1)}}{R^n}. \tag{119}$$

In that case, assuming again the power law for time dependence of the scale factor, one gets

$$w_{eff} = -1 + \frac{2(n+2)}{3(2n+1)(n+1)}. \tag{120}$$

The condition for accelerated expansion $w_{eff} < -1/3$ for the two above considered models is transformed into the form

$$n^2 \mp n - 1 > 0,$$

where the signs $\mp$ respond to the first and the second examples respectively. In particular, in the second case, for $n = 1$ we find that $w_{eff} = -2/3$. We remark that in such case the positive values of n imply the presence of terms inversely proportional to R, unlike the above considered case.

B. Braneworld Cosmology

All models explaining the accelerated expansion of Universe share one common property: they involve one or another way to weaken the gravity. It is the negative pressure in SCM and in the scalar field models, transformation of the universal gravity law in the modified gravitation model or voids in the models with inhomogeneities. An original approach to suppress the gravity is realized in the Braneworld scenario [89]. According to latter we live on a three-dimensional brane, which is embedded in the volume of higher dimension (in the simplest case it is four-dimensional). All material fields are restricted to the brane, while the gravitation penetrates both the brane and whole embedding volume (see Lecture notes by R.Maartens [94]). Higher dimension of the space where the gravity acts, results in its suppression ($F \propto R^{-(D-1)}$, where D is the dimension of the space).

According such scenario the equation of motion for the scalar field on the brane reads

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0. \tag{121}$$

where

$$H^2 = \frac{8\pi}{3{M_{Pl}}^2}\rho\left(1 + \frac{\rho}{2\sigma}\right) + \frac{\Lambda_4}{3} + \frac{\varepsilon}{a^4}; \tag{122a}$$

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi). \tag{122b}$$

Here ε is the integration constant, which transforms the volume gravity to the brane restricted one. The brane tension σ provides connection between the for-dimensional Planck mass M_{Pl} and five-dimensional one $M_{Pl}^{(5)}$:

$$M_{Pl} = \sqrt{\frac{3}{4\pi}}\left(\frac{{M_{Pl}^{(5)}}^3}{\sqrt{\sigma}}\right). \tag{123}$$

The tension σ also links the four-dimensional cosmological constant Λ_4 on the brane with the five-dimensional volume one Λ_b by:

$$\Lambda_4 = \frac{4\pi}{{M_{Pl}^{(5)}}^3}\left(\Lambda_b + \frac{4\pi}{3{M_{Pl}^{(5)}}^3}\sigma^2\right). \tag{124}$$

Remark, that the equations (122a - 122b) contain an additional term $\propto \rho^2/\sigma$, whose presence is connected with the conditions on the brane-volume boundary. The latter term is responsible for dynamical enhancement of the decay experienced by the scalar field while it rolls down the potential. In such a case inflation regime can be realized even for those potentials which are too steep to satisfy the slow-roll inflation conditions in the standard approach. Indeed, the slow-roll parameters in the braneworld models in the limit $V/\sigma \gg 1$ read:

$$\varepsilon \simeq 4\varepsilon_{FRW}\left(\frac{V}{\sigma}\right)^{-1}; \quad \eta \simeq 2\eta_{FRW}\left(\frac{V}{\sigma}\right)^{-1}. \tag{125}$$

It follows now that the slow-roll inflation condition ($\varepsilon, \eta \ll 1$) is easier satisfied than in the FRW cosmology under the condition $V/\sigma \gg 1$. The inflation thus can take place for very steep quintessence potentials, such as $V \propto e^{-\lambda\varphi}$, $V \propto \varphi^{-\alpha}$ and others. This in turn let us hope that inflation and quintessence can be generated by the same scalar field.

A radically different approach to provide the accelerated expansion of Universe was proposed in papers [90], [91], [92].

The sharp distinction of the DDG model from the RS one is in the fact that both the volume cosmological constant and brane tension are set to zero, while a curvature term is introduced into the brane action. The theory is based on the following action

$$S = {M_{Pl}^{(5)}}^3\int_{bulk} R + {M_{Pl}}^2\int_{brane} R + \int_{brane} L_{matter}. \tag{126}$$

The physical sense of the term $\int_{brane} R$ is unclear. Probably, it accounts for quantum effects caused by material fields, which lead to the same term in the Einstein action, as was first noted by A.D. Sakharov in its induced gravity theory[93].

The Hubble parameter in the DDG braneworld takes the form

$$H=\sqrt{\frac{8\pi G\rho_m}{3}+\frac{1}{l_c^2}}+\frac{1}{l_c}, \tag{127}$$

where $l_c={M_{Pl}}^2/{M_{Pl}^{(5)}}^3$ is a new length scale, defined by the 4D Planck mass M_{Pl} and 5D one $M_{Pl}^{(5)}$. An important property of such a model is that the accelerated expansion of Universe does require presence of dark energy. Instead, since gravity becomes five dimensional on length scales $R>l_c=2H^{-1}(1-\Omega_m)^{-1}$, one finds that the expansion of the Universe is modified during late times instead of early times as in RS model.

More general class of braneworld models, which includes both RS and DDG models as particular cases, is described by the action

$$S={M_{Pl}^{(5)}}^3\int_{bulk}(R-2\Lambda_b)+\int_{brane}({M_{Pl}}^2R-2\sigma)+\int_{brane}L_{matter}. \tag{128}$$

For $\sigma=\Lambda_b=0$ the action (128) transforms into the DDG model action (126), and for $m=0$ it reduces to RS-model.

As was shown in [92], the action (128) describes the Universe, where the accelerated expansion era occurs on later evolution stages with the Hubble parameter

$$\frac{H^2(z)}{H_0^2}=\Omega_m(1+z)^3+\Omega_\sigma+2\Omega_l\mp2\sqrt{\Omega_l}\sqrt{\Omega_m(1+z)^3+\Omega_\sigma+\Omega_l+\Omega_{\Lambda_b}}, \tag{129}$$

where

$$\Omega_l=\frac{1}{l_c^2H_0^2},\ \Omega_m=\frac{\rho_{0m}}{3{M_{Pl}}^2H_0^2},\ \Omega_\sigma=\frac{\sigma}{3{M_{Pl}}^2H_0^2},\ \Omega_{\Lambda_b}=-\frac{\Lambda_b}{6H_0^2}. \tag{130}$$

Signs $\mp$ correspond to two possible ways to embed a brane in to the volume. As it was in DDG model, $l_c\sim H_0^{-1}$ if $M_{Pl}^{(5)}\sim 100MeV$. On shorter length scales $r\ll l_c$ and on early times we recover the general relativity, while on large length scales $r\gg l_c$ and long time periods the brane effects start to be important. Indeed, setting $M_{Pl}^{(5)}=0$ ($\Omega_l=0$) the equation (129) reduces to the ΛCDM model:

$$\frac{H^2(z)}{H_0^2}=\Omega_m(1+z)^3+\Omega_\sigma. \tag{131}$$

while for the case $\sigma=\Lambda_b=0$ the relation (129) reproduces the DDG model. An important feature of the action (128) is the fact that it generates the effective state equation $w_{eff}\le -1$. It can be easily seen [92] from the expression for current value of the effective parameter in the state equation

$$w_0=\frac{2q_0-1}{3\,(1-\Omega_m)}=-1\pm\frac{\Omega_m}{1-\Omega_m}\sqrt{\frac{\Omega_l}{\Omega_m+\Omega_\sigma+\Omega_l+\Omega_{\Lambda_b}}}. \tag{132}$$

Taking the lower sign, one gets $w_0<-1$.

It is worth noting that in the latter model the accelerated Universe expansion phase is the transient phenomenon, which comes to end when the Universe returns to the matter dominated phase.

C. MOND

The so-called modified Newtonian dynamics (MOND) is sometimes considered as an alternative to the DM (dark matter) concept. MOND represents such modification of Newton physics, which allows to explain the flat rotational curves for galaxies without attributing to any assumptions on DM. MOND assumes that the second Newton law $F=ma$ must be modified for sufficiently low accelerations ($a\ll a_0$) so that

$$\vec F=m\vec a\mu(a/a_0). \tag{133}$$

where $\mu(x)=x$ if $x\ll1$, and $\mu(x)=1$ if $x\gg1$. It is easy to see that such law leads to the modification of the traditional formula for the gravitational acceleration $\vec F=m\vec g_N$ ($g_N=GM/r^2$). Relation between the "correct" acceleration and traditional Newtonian one reads

$$a=\sqrt{a_0g_N}. \tag{134}$$

For a rotating point mass one gets $a = v^2/r$ (this purely kinematic relation is independent on choice of the dynamic model). But here a stands already for "correct" acceleration. It follows that

$$v^4 = GMa_0. \tag{135}$$

i.e. for sufficiently small accelerations the rotational curves for isolated body of mass M do not depend on radial distance r, where the velocity is measured. In other words, the considered theory predicts not only flat rotational curves but also the fact that individual halo, associated with a galaxy, has infinite extension. This prediction may cause a serious problem for MOND, as recent observations of galaxy-galaxy lensing support the result that the maximum halo extension is about 0.5 Mpc. The value a_0, needed to explain the observations

$$a_0 \sim 10^{-8}\, cm/s^2. \tag{136}$$

is of the same order as cH_0. It supports the hypothesis that MOND "can reflect the influence of cosmology on local particle dynamics".

Although the results of MOND agree very well with the observations of individual galaxies, it is unclear whether they will be as well successful for description of cluster structure, where strong gravitational lensing points out considerably denser mass concentration in the cluster center than was predicted by MOND. Another difficulty connected with MOND, is the fact that it is quite problematic to incorporate it in more general relativistic gravity theory. Presently it is not clear what predictions are given by the MOND-like theories for complicated gravity effects like the gravitational lensing.

VIII. DYNAMICS OF UNIVERSE WITH INTERACTION IN THE DARK SECTOR

SCM considers dark matter and dark energy as independent components of energy budget of Universe. The dark energy postulated in form of cosmological constant, introduced as early as by Albert Einstein in order to make possible the creation of stationary Universe model. The assumed absence of interaction between the two components means that the energy densities for each component obey independent conservation equations

$$\dot{\rho}_i + 3H\left(\rho_i + p_i\right) = 0.$$

Coupling between the two components leads to modified evolution of Universe. In particular, the energy density for non-relativistic component will not evolve according the law a^{-3}, and the dark energy density (in form of the cosmological constant) will not remain constant any more. From one hand, such modification of the theory opens for us new possibilities for solution of principal problems in cosmology. So, for example, solution of the coincidence problem[3] reduces to appropriate choice of interaction parameters, which can provide satisfaction of the condition

$$\frac{\Omega_{de}}{\Omega_{dm}} = \mathrm{O}(1),$$

in the present time consistently with the condition $\ddot{a} > 0$. From the other hand, introduction of the interaction will modify the relations between the observable parameters. In particular, the modification will affect the fundamental relation between the photometric distance and the redshift, which the evidence of accelerated expansion of Universe is mainly based on. It imposes strict limitations both on the form of interaction and its parameters.

Taking into account that dark matter and dark energy represent dominant components in the Universe, then from point of view of field theory it is naturally to consider interaction between them [167, 169–174, 182, 198]. Appropriate interaction between the dark matter and dark energy can facilitate solution of the coincidence problem [175–177]. Non-minimal coupling in the dark sector can considerably affect the history of cosmological expansion of Universe and evolution of density fluctuations, thus modifying the rate of the cosmological structure growth. The possibility of dark matter and dark energy to interact with each other is widely discussed in recent literature [167, 169–177, 182, 198]. Different and independent data of many observations, such as Wilkinson Microwave Anisotropy Probe, SNe Ia and BAO, were specially analyzed in order to investigate the limitations on the intensity and form of the interaction in the dark sector. Some researchers also suggest [192], that dynamical equilibrium of collapsing structures, such as clusters, will essentially depend on form and sign of the interaction between dark matter and dark energy.

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[3] the problem is that during all the Universe history the two densities decay by different laws, so it is necessary to impose very strict limitations on their values in early Universe in order to make them be of comparable order nowadays

A. Model of Universe with time dependent cosmological constant

The simplest example of the model with interacting DM and DE is the cosmological model with decaying vacuum. Actually the Λ(t)CDM cosmology represents one of the cases where parameter w for dark energy equals to -1.

This model is based on the assumption that the dark energy is nothing that physical vacuum, and energy density of the latter is calculated on the curved space background with subtraction of renormalized energy density of physical vacuum in the flat space [193]. The resulting effective energy density of physical vacuum depends on space-time curvature and decays from high initial values in early Universe (at strong curvature) to almost zero magnitude in the present time.

Due to the Bianchi identity, the decay of vacuum must be accompanied by creation or mass increase of dark energy particles, which is common property of the decaying vacuum models, or in more general case, for the models with interacting dark matter and dark energy.

Now let us give the formulation of the Λ(t)CDM. In the present section we will use the system of units where the reduced Planck mass equals to unity: $M_{Pl} = (8\pi G)^{-1/2} = 1$.

For the case of flat Universe described by FRW-metric, the first Friedmann (13) and the conservation equation can be presented in the form

$$\rho_{tot} = 3H^2, \quad \dot\rho_{tot} + 3H(\rho_{tot} + p_{tot}) = 0,$$

where $\rho_{tot} = \rho_m + \rho_\Lambda$, and taking into account that $p_m = 0$ and $p_\Lambda = -\rho_\Lambda$, the conservation equation takes the form:

$$\dot\rho_m + 3H\rho_m = -\dot\rho_\Lambda. \tag{137}$$

The right hand side of the latter contains an additional term which plays role of a source — it is the decaying cosmological constant. The problem with such approach is in fact that we end up with the same number of equations, but acquire additional unknown function $\Lambda(t)$. The above described approach to define the physical vacuum density, though intuitively clear, faces certain principal difficulties [168]. That is why a phenomenological approach prevails in the literature. Below we present a simple but exactly solvable model. Consider the case when Λ depends on time as [74]:

$$\Lambda = \sigma H. \tag{138}$$

It is interesting to note that with the choice $\sigma \approx m_\pi^3$ (m_π is the energy scale of the QCD vacuum condensation) the relation (138) gives the value of Λ which is very close to the observed one.

For the considered components the first Friedmann equation takes the form

$$\rho_\gamma + \Lambda = 3H^2. \tag{139}$$

The equations (137) and (139) together with the conservation equation

$$p_\gamma = w\rho_\gamma \equiv (\gamma - 1)\,\rho_\gamma,$$

and the decay law for the cosmological constant (137) completely determine the scale factor time evolution. Combining those equation, one obtains the evolution equation in the following form

$$2\dot H + 3\gamma H^2 - \sigma\gamma H = 0.$$

Under condition $H > 0$ the solution for the latter equation reads

$$a(t) = C[\exp(\sigma\gamma t/2) - 1]^{\frac{2}{3\gamma}},$$

where C is one of the two integration constants (recall that the equation for the scale factor is of second order). The second integration constant is determined from the condition $a(t = 0) = 0$. Using such solution, one can find the densities of matter (radiation) and time-dependent cosmological constant

$$\rho_\gamma = \frac{\sigma^2}{3}\left(\frac{C}{a}\right)^{\frac{3}{2}\gamma}\left[1 + \left(\frac{C}{a}\right)^{\frac{3}{2}\gamma}\right]; \tag{140}$$

$$\Lambda = \frac{\sigma^2}{3}\left[1 + \left(\frac{C}{a}\right)^{\frac{3}{2}\gamma}\right]. \tag{141}$$

Let us now analyze the history of Universe expansion in the scenario under consideration. In the radiation-dominated epoch $\gamma = w + 1 = 4/3$ and, therefore,

$$a(t) = C\left[e^{\frac{2}{3}\sigma t} - 1\right]^{\frac{1}{2}}.$$

For small values of time ($\sigma t \ll 1$) and we recover the well-known dependence $a \propto t^{1/2}$,

$$a(t) = C\sqrt{\frac{2}{3}}\sigma t^{\frac{1}{2}}.$$

The densities ρ_γ(140) and Λ (141) in the radiation dominated epoch transform into

$$\rho_\gamma \to \rho_r = \frac{\sigma^2 C^4}{3}\frac{1}{a^4} + \frac{\sigma^2 C^2}{3}\frac{1}{a^2};$$

$$\Lambda = \frac{\sigma^2}{3} + \frac{\sigma^2 C^2}{3}\frac{1}{a^2}.$$

In the linit $a \to 0$ $(t \to 0)$ one has

$$\rho_r = \frac{\sigma^2 C^4}{3}\frac{1}{a^4} = \frac{3}{4t^2};$$

$$\Lambda = \frac{\sigma^2 C^2}{3}\frac{1}{a^2} = \frac{\sigma}{2t}.$$

Now consider the matter dominated era. In that case $\gamma = w + 1 = 1$ and

$$a(t) = C\left[e^{\frac{1}{2}\sigma t} - 1\right]^{\frac{2}{3}}.$$

For $\sigma t \ll 1$ we reproduce the standard law for matter evolution

$$a(t) = C\left(\frac{\sigma}{2}\right)^{\frac{2}{3}} t^{\frac{2}{3}}.$$

For the densities ρ_γ and Λ in the matter dominated era one finds

$$\rho_\gamma \to \rho_m = \frac{\sigma^2 C^3}{3}\frac{1}{a^3} + \frac{\sigma^2 C^{3/2}}{3}\frac{1}{a^{3/2}};$$

$$\Lambda = \frac{\sigma^2}{3} + \frac{\sigma^2 C^{3/2}}{3}\frac{1}{a^{3/2}}.$$

Note that in the limit of long times ($\sigma t \gg 1$) the scale factor growth exponentially as

$$a(t) = Ce^{\frac{\sigma}{3}t}.$$

In the same limit the matter density tends to zero, and the time-dependent function $\Lambda(t)$ transforms into the "real" cosmological constant.

In the matter dominated epoch the Friedmann equation (139) can be presented in the form

$$H(z) = H_0\left[1 - \Omega_{m0} + \Omega_{m0}(1+z)^{3/2}\right].$$

This expression can be used for calculation of the deceleration parameter $\left(q = \frac{1+z}{H}\frac{dH}{dz} - 1\right)$,

$$q(z) = \frac{\frac{3}{2}\Omega_{m0}(1+z)^{3/2}}{1 - \Omega_{m0} + \Omega_{m0}(1+z)^{3/2}} - 1. \tag{142}$$

One thus finds for the current value of the deceleration parameter

$$q(z=0)=\frac{3}{2}\Omega_{m0}-1.$$

Therefore the accelerated expansion of Universe occurs under the following condition

$$\Omega_{m0}<\frac{2}{3},$$

This condition is in fact satisfied for the observed value $\Omega_{m0}\approx 0.23$. From (142) it follows that the transition from the decelerated expansion to the accelerated one took place at

$$z^{*}=\left[2\frac{1-\Omega_{m0}}{\Omega_{m0}}\right]^{2/3}-1.$$

This value ($z^{*}\approx 1.2$) exceeds (being however of the same order O (1)) the corresponding value for SCM ($z^{*}\approx 0.75$), which is the result of the matter production in the vacuum decay process.

Therefore the model based on the time-dependent cosmological constant, where vacuum density linearly depends on Hubble parameter, appears to be quite competitive. It sufficiently accurately reproduces the "canonic" results, relative both to the radiation dominated and matter dominated eras. The present Universe expansion is accelerated according to the model under consideration. Verification of the model with the latest observational data obtained for SN1a leads to the results (e.g., $0.27\leq\Omega_{m0}\leq 0.37$), that agree very well with the currently accepted estimates for the accelerated Universe expansion parameters.

B. Interacting dark matter and dark energy

One of the most interesting properties of dark matter is its possible interaction with dark energy. Although the most realistic models (SCM in particular) postulate that the dark matter and dark energy are uncoupled, there is no serious evidence to consider such assumption as a principle. In the present time many active researches are provided to investigate the possible consequences of such interaction [163, 166, 167, 169–177, 182, 192, 198]. As we have mentioned above, the interaction of dark components can at least soften some sharp cosmological problems, such as the coincidence problem for example. The dark energy density is approximately three times as high as the dark matter one. Such coincidence could be explained if dark matter was somehow sensitive to dark energy.

Remark that the possibility of interaction between the dark energy in form of scalar field and dark matter lies in the basis of warm inflation model. Unlike the cold inflation scenario, the former does not assume that the scalar field in the inflation period is isolated (uncoupled) from other fields. That is why instead of overcooled Universe in the inflation period, in the Universe model under discussion a certain quantity of radiation is always conserved, which is sufficiently remarkable to manifest in post-inflation dynamics.

Interaction of the scalar field quantum, responsible for the inflation start – the inflaton – with other fields imply that its master equation contains the terms describing the process of energy dissipation from inflaton system to other particles. Barera and Fang [187] initially assumed that consistent description of the inflaton field with energy dissipation requires the master equation in form of the Langeven equation, where the fluctuation-dissipation relation is present, which uniquely links the field fluctuations and dissipated energy. Such equation lies in the the basis of description of the warm inflation process.

Interaction between the components in the Universe must be introduced in such a way that preserves the covariance of the energy-momentum tensor $T^{\mu\nu}_{(tot)\ ;\nu}=0$, therefore $T^{\mu\nu}_{DM\ ;\nu}=-T^{\mu\nu}_{DE\ ;\nu}\neq 0$, where u_ν is the 4-velocity. The conservation equations in that case take the form:

$$u_\nu T^{\mu\nu}_{DM\ ;\mu}=-u_\nu T^{\mu\nu}_{DE\ ;\mu}=-Q. \tag{143}$$

For FRW-metric the equations (143) transform to:

$$\dot\rho_m+3H\rho_m=Q, \tag{144}$$
$$\dot\rho_{DE}+3H\rho_{DE}(1+w_{DE})=-Q, \tag{145}$$

where ρ_m and ρ_{DE} are densities of dark matter and dark energy respectively, w_{DE} is the parameter of state equation for for dark energy, $H\equiv\dot a/a$ is the Hubble parameter.

$$Q\left\{\begin{array}{l}>0\\<0\end{array}\right.\quad\Rightarrow\quad\text{energy transits}\left\{\begin{array}{l}\mathrm{DE}\rightarrow\mathrm{DM};\\\mathrm{DM}\rightarrow\mathrm{DE}\end{array}\right.$$

If $Q<0$, then dark matter continuously decays into dark energy, if $Q>0$ then vice versa. Note that the equations (144) and (145) obey the conservation equation:

$$\dot{\rho}_{tot}+3H\rho_{tot}(\rho_{tot}+p_{tot})=0, \tag{146}$$

where $\rho_{tot}=\rho_{DE}+\rho_m$ is the total energy density.

The interaction between the dark matter and dark energy is effectively equivalent to modification of the state equation for the interacting components. Indeed, the equations (144) can be transformed to the standard form of the conservation law written for uncoupled components:

$$\begin{array}{l}\dot{\rho}_{DE}+3H\rho_{DE}(1+w_{DE,eff})=0\\ \dot{\rho}_{DM}+3H\rho_{DM,eff}=0,\end{array},$$

where

$$w_{DE,eff}=w_{DE}-\frac{Q}{3H\rho_{DE}};\quad w_{DM,eff}=\frac{Q}{3H\rho_{DM}}, \tag{147}$$

play role of effective state equations for dark energy and dark matter respectively.

As we do not know the nature of both the dark energy and dark matter, we cannot derive the coupling Q from the first principles [185]. However it is clear from dimension analysis that this quantity must depend on the energy density of one of the dark component, or a combination of both components with dimension of energy density, times the quantity of inverse time dimension. For the latter it is natural to take the Hubble parameter H.

Different forms of Q the most often used in the literature, are the following:

$$Q=3H\gamma\rho_m,\quad Q=3H\gamma\rho_{DE},\quad Q=3H\gamma(\rho_m+\rho_{DE}). \tag{148}$$

For example we consider the simplest case for such model

$$\begin{array}{l}\dot{\rho}_m+3H\rho_m=\delta H\rho_m,\\ \dot{\rho}_{DE}+3H\left(\rho_{DE}+p_{DE}\right)=-\delta H\rho_m,\end{array}$$

where δ is the dimensionless coupling constant.

Integration of the latter equations yields

$$\begin{array}{l}\rho_{DE}=\rho_{DE0}a^{-\left[3(1+w_{DE})+\delta\right]};\\ \rho_m=\frac{-\delta\rho_{DE0}}{3w_{DE}+\delta}a^{-\left[3(1+w_{DE})+\delta\right]}+\left(\rho_{m0}+\frac{\delta\rho_{DE0}}{3w_{DE}+\delta}\right)a^{-3}.\end{array} \tag{149}$$

Inserting the obtained densities into the first Friedmann equation (13) and transforming from scale factor to redshift, one gets for $H^2(z)$ the following

$$H^2=\frac{(1+z)^3H_0^2}{3(3w_{DE}+\delta)}\left[3w_{DE}\Omega_{DE}(1+z)^{(3w_{DE}+\delta)}+\Omega_m(3w_{DE}+\delta)\right]. \tag{150}$$

Note that in the considered case the deceleration parameter $q(z)$ can be easily determined by the following

$$q(z)=\frac{1+z}{2H^2}\frac{dH^2}{dz}-1,$$

and direct substitution of (150) results in explicit dependence of deceleration parameter on the redshift:

$$q(z)=-1+\frac{3}{2}\frac{w_{DE}\Omega_{DE}(3(1+w_{DE})+\delta)(1+z)^{(3w_{DE}+\delta)}+\Omega_m+\delta/w_{DE}}{3w_{DE}\Omega_{DE}(1+z)^{(3w_{DE}+\delta)}+\Omega_m+\delta/w_{DE}}. \tag{151}$$

With $w_{DE}=-1$ and $\delta=0$, this expression coincides with the deceleration parameter value obtained in SCM. Note that in the considered model appropriate choice of the coupling parameter value δ can essentially soften or even get rid of the compatibility problem for the densities of dark matter and dark energy. Indeed, using the densities (149), one finds the following relation $R=\rho_m/\rho_{DE}$:

$$R=-\frac{\delta}{3w_{DE}+\delta}+\left(R_0+\frac{\delta}{3w_{DE}+\delta}\right)a^{3w_{DE}+\delta},$$

where $R_0 = \rho_{m0}/\rho_{DE0}$ is the ratio of the densities in the present time. In SCM it is known to be $R \sim a^{-3}$, which differs from the considered model on the δ in exponent.

Let us now consider the inverse problem, and instead of interaction we postulate the ratio

$$\frac{\rho_m}{\rho_{DE}} = f(a), \tag{152}$$

where $f(a)$ is arbitrary differentiable function of scale factor. Thus one obtains:

$$\rho_m = \rho_{DE} f(a), \tag{153}$$
$$\rho_{DE} = \frac{\rho_m}{f(a)}, \tag{154}$$
$$\dot\rho_m = \dot\rho_{DE} f + \rho_{DE} f' \dot a, \tag{155}$$
$$\dot\rho_{DE} = \frac{\dot\rho_m}{f} - \frac{\rho_m \dot a f'}{f^2}. \tag{156}$$

Inserting the expressions (155) and (153) into the equation (144) we obtain:

$$\dot\rho_{DE} f + \rho_{DE} f' \dot a + 3H\rho_{DE} f = Q, \tag{157}$$

where prime denotes the differentiation with respect to the scale factor. Using here the expression for $\dot\rho_{DE}$, obtained from (145), one gets:

$$Q = \frac{H\rho_{DE} f}{1+f}\left(\frac{f' a}{f} - 3w_{DE}\right). \tag{158}$$

The first Friedmann equation takes on the form:

$$3H^2 = \rho_{DE} + \rho_m = \rho_{cr}, \tag{159}$$

where ρ_{cr} is the critical density. Accordingly one can write down that

$$\Omega_{DE} = \frac{\rho_{DE}}{\rho_{cr}} = \frac{1}{1+f}; \tag{160}$$
$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{f}{1+f}. \tag{161}$$

Now, substituting (161) into (158), we finally obtain the expression for Q:

$$Q = H\rho_{DE}\Omega_m\left(\frac{f' a}{f} - 3w_{DE}\right) = H\rho_m\Omega_{DE}\left(\frac{f' a}{f} - 3w_{DE}\right). \tag{162}$$

It is worth noting that in the case when $f(a) \propto a^{\xi}$ we obtain the expression

$$Q = H\rho_m\Omega_{DE}\left(\xi - 3w_{DE}\right) = H\rho_{DE}\Omega_m\left(\xi - 3w_{DE}\right). \tag{163}$$

From the equation (146) it follows that:

$$\frac{d\ln\rho_{tot}}{d\ln a} = -3(1 + w_{eff}), \tag{164}$$

where

$$w_{eff} = \frac{p_{tot}}{\rho_{tot}} = \frac{\rho_{DE} w_{DE}}{\rho_{DE} + \rho_m} = \frac{w_{DE}}{1+f} = \Omega_{DE} w_{DE}, \tag{165}$$

and therefore

$$\frac{d\ln\rho_{tot}}{d\ln a} = -3(1 + \Omega_{DE} w_{DE}). \tag{166}$$

From the latter equation we obtain:

$$\rho_{tot} = Ca^{-3}\exp\left(-\int 3\Omega_{DE} w_{DE}\, d\ln a\right) \tag{167}$$

where C is the integration constant determined from the requirement that $\rho_{tot}(a=1)=\rho_{tot,0}=3H_0^2/(8\pi G)$. Using the expression for ρ_{tot}, one can easily obtain from the Friedmann equations the relation for $E\equiv H/H_0$, which is used to verify cosmological models and establish restrictions on cosmological parameters. Expression for $\rho_{_{DE}}=\Omega_{_{DE}}\rho_{tot}$ and $\rho_m=\Omega_m\rho_{tot}$ can also easily obtained from the equations (160) and (167).

At last it is worth noting that one can obtain from the equation (152),

$$f_0=f(a=1)=\frac{\rho_{m0}}{\rho_{_{DE0}}}=\frac{\Omega_{m0}}{1-\Omega_{m0}}=\frac{1}{\Omega_{_{DE0}}}-1. \tag{168}$$

Let it be now

$$f(a)=f_0\,a^{\xi}, \tag{169}$$

where ξ is a constant and f_0 is defined above. Substituting the equation (169) into the equation (167) and requiring that $\rho_{tot}(a=1)=\rho_{tot,0}$, we can determine the integration constant and find that

$$\rho_{tot}=\rho_{tot,0}\,a^{-3}\left[\,\Omega_{m0}+(1-\Omega_{m0})\,a^{\xi}\,\right]^{-3w_{eff}/\,\xi}. \tag{170}$$

Inserting this expression into the Friedmann equation, one finds:

$$\begin{aligned}E^2=\frac{H^2}{H_0^2}&=a^{-3}\left[\,\Omega_{m0}+(1-\Omega_{m0})\,a^{\xi}\,\right]^{-3w_X/\,\xi}\\ &=(1+z)^3\left[\,\Omega_{m0}+(1-\Omega_{m0})\,(1+z)^{-\xi}\,\right]^{-3w_X/\,\xi}.\end{aligned} \tag{171}$$

Using the above obtained formulae, it is easy to find the following expression for the deceleration parameter in the considered model

$$q=1+\frac{3}{2}\left(w_{_{DE}}\Omega_{_{DE}}+w_m\Omega_m\right), \tag{172}$$

where relative densities $\Omega_{_{DE}}$, $\Omega_{_{DM}}$ has the following form

$$\Omega_{_{DE}}=\frac{\Omega_{_{DE0}}}{\Omega_{_{DE0}}+(1-\Omega_{_{DE0}})a^{\xi}},\quad \Omega_{_{DM}}=\frac{(1-\Omega_{_{DE0}})a^{\xi}}{\Omega_{_{DE0}}+(1-\Omega_{_{DE0}})a^{\xi}}, \tag{173}$$

and we finally obtain

$$q=\frac{1}{2}+\frac{3}{2}\frac{w_{_{DE}}\Omega_{_{DE0}}(1+z)^{\xi}}{1-\Omega_{_{DE0}}+\Omega_{_{DE0}}(1+z)^{\xi}}, \tag{174}$$

As the dark energy density is not constant in the considered model, unlike the SCM, it facilitates solution of the coincidence problem under condition $\xi<3$. In the model under consideration the accelerated expansion phase starts at the redshift z^*, which is defined by the relation $q(z^*)=0$, and it equals to

$$z^*=\left(\frac{1-\Omega_{_{DE0}}}{(1+3w_{_{DE}})\Omega_{_{DE0}}}\right)^{\frac{1}{\xi}}-1. \tag{175}$$

This relation naturally generalizes the expression (58), obtained for SCM. The difference of the value (175) from the corresponding one for SCM depends on magnitude of the distinction of the parameters ξ and $w_{_{DE}}$ from 3 and -1 respectively. In the case of zero difference the quantities (58) and (175) coincide. Note that at $z\to\infty$ Universe expanded with deceleration $q\to\frac{1}{2}$, the case $z\to-1$ responds to $q\to\frac{1}{2}-\frac{2}{3}w_{_{DE}}$. Therefore, as it was expected, dynamics of the considered model is asymptotically (at $z\to\infty$ and $z\to-1$) identical to the case of two-component Universe, filled by non-relativistic matter and dark energy with the state equation $p=w_{_{DE}}\rho$.

C. Cosmological models with the new type of interaction

In the present subsection we consider one more type of interaction Q, [75] which change its sign, i.e. direction of energy transfer, when the expansion changes from decelerated regime to accelerated one and vice versa.

Some recently appeared papers [79] make attempt to determine both the very possibility of interaction in the dark sector and specify its form and sign, basing solely on analysis o observational data. The analysis splits all available set of redshift data z into some parts, within which the function $\delta(z) = Q/(3H)$ is considered to be constant. This analysis allowed to establish that the function $\delta(z)$ the most likely takes zero values ($\delta = 0$) in the interval of redshifts $0.45 \lesssim z \lesssim 0.9$. It turns out that this remarkable result produces new problems. Indeed, as we have already mentioned, the researchers mostly consider the interaction of the form (148): $Q = 3H\gamma\rho_m,\ Q = 3H\gamma\rho_{_{DE}},\ Q = 3H\gamma(\rho_m + \rho_{_{DE}})$ and for the model under consideration it is always either positive or negative defined, and therefore it cannot change its sign. Sign changes are possible only in the case when $\gamma = f(t)$, which can change the sign of Q or $Q = 3H(\alpha\rho_m + \beta\rho_{_{DE}})$, where α and β have opposite signs.

As authors of [79] mention, solution of such problem requires to introduce a new type of interaction, which can change its sign during the evolution of Universe.

An interaction Q of such type was proposed in the paper [75], where its cosmological consequences were considered. As it was noted in the above cited paper, the redshift interval, where the function $\delta(z)$ must change its sign, includes the transition point when the Universe expansion stopped to decelerate and started the acceleration (see (58)). Therefore the simplest type of the interaction which can explain the above mentioned property is the case when the source Q is proportional to the deceleration parameter q :

$$Q = q(\alpha\dot{\rho} + 3\beta H\rho) \tag{176}$$

where α and β are dimensionless constants, and the sign of Q will change with the transition of universe from the decelerated expansion stage ($q > 0$) to the accelerated one ($q < 0$). The authors also consider the case

$$Q = q(\alpha\dot{\rho}_m + 3\beta H\rho_m), \tag{177}$$
$$Q = q(\alpha\dot{\rho}_{tot} + 3\beta H\rho_{tot}), \tag{178}$$
$$Q = q(\alpha\dot{\rho}_{_{DE}} + 3\beta H\rho_{_{DE}}). \tag{179}$$

The paper [78] considers a model of Universe with decaying cosmological constant

$$\dot{\rho}_\Lambda = -Q\,.$$

The Friedmann and Raychaudhuri equation take thus the form

$$H^2 = \frac{\kappa^2}{3}\rho_{tot} = \frac{\kappa^2}{3}(\rho_\Lambda + \rho_m)\,, \tag{180}$$
$$\dot{H} = -\frac{\kappa^2}{2}(\rho_{tot} + p_{tot}) = -\frac{\kappa^2}{2}\rho_m, \tag{181}$$

where $\kappa^2 \equiv 8\pi G$. Following the paper [78], in the succeeding subsections we consider cosmological models with interaction of the type (177)-(179).

1. Case $Q = q(\alpha\dot{\rho}_m + 3\beta H\rho_m)$

For the beginning we consider the case when the interaction takes the form (177) and insert such expression into the conservation equation (144), resulting in the following

$$\dot{\rho}_m = \frac{\beta q - 1}{1 - \alpha q}\cdot 3H\rho_m\,. \tag{182}$$

Substituting the obtained expression into the equation (177), we finally get

$$Q = \frac{\beta - \alpha}{1 - \alpha q}\cdot 3qH\rho_m. \tag{183}$$

From the equation (181) one obtains

$$\rho_m = -\frac{2}{\kappa^2}\dot{H}. \tag{184}$$

Inserting it into the equation (182) one finds that

$$\ddot{H} = \frac{\beta q - 1}{1 - \alpha q}\cdot 3H\dot{H}\,, \tag{185}$$

Thus we obtained the second order differential equation for the function $H(t)$. Now transform from the time derivative to differentiation with respect to the scale factor (denoted by the prime $'$), then the equation (185) takes on the form

$$aH'' + \frac{a}{H}H'^2 + H' = \frac{\beta q - 1}{1 - \alpha q} \cdot 3H' \,. \tag{186}$$

This expression represents a second order differential expression for the function $H(a)$. Note that the deceleration parameter

$$q = -1 - \frac{\dot{H}}{H^2} = -1 - \frac{a}{H}H' \,,$$

is also function of H and H' and, except the case $\alpha \neq 0$, the equation has no exact solution and it represents a transcendental differential equation of second order. That is why we consider solely the case $\alpha = 0$. Thus the interaction (177) takes the form

$$Q = 3\beta q H \rho_m.$$

With $\alpha = 0$ the solution (186) can be presented in the form

$$H(a) = C_{12}\left[3C_{11}(1+\beta) - (2+3\beta)\, a^{-3(1+\beta)}\right]^{1/(2+3\beta)}, \tag{187}$$

where C_{11} and C_{12} are the integration constants determined below. We find the relative density of dark matter as the following

$$\Omega_m \equiv \frac{\kappa^2\rho_m}{3H^2} = -\frac{2\dot{H}}{3H^2} = -\frac{2aH'}{3H} \,. \tag{188}$$

Inserting the equation (187) into (188), one gets

$$\Omega_m = \frac{2\,(1+\beta)}{2 + 3\beta - 3C_{11}\,(1+\beta)\; a^{3(1+\beta)}} \,. \tag{189}$$

With the requirements $\Omega_m(a=1) = \Omega_{m0}$ and $H(a=1) = H_0$, the integration constants take the form

$$C_{11} = \frac{\Omega_{m0}(2+3\beta) - 2(1+\beta)}{3\Omega_{m0}(1+\beta)}, \tag{190}$$

$$C_{12} = H_0\left[3C_{11}(1+\beta) - (2+3\beta)\right]^{-1/(2+3\beta)} \,. \tag{191}$$

Substitution of the expressions (190) and (191) into the equation (187) finally gives the result

$$E \equiv \frac{H}{H_0} = \left\{1 - \frac{2+3\beta}{2(1+\beta)}\,\Omega_{m0}\left[1 - (1+z)^{3(1+\beta)}\right]\right\}^{1/(2+3\beta)} \,. \tag{192}$$

The model contains two free parameters Ω_{m0} and β. We note that if $\beta = 0$, then the equation (192) reduces to $E(z) = \left[\Omega_{m0}(1+z)^3 + (1-\Omega_{m0})\right]^{1/2}$, which is equivalent to ΛCDM model. Using the relation

$$q(z) = -\frac{(1+z)}{E(z)}\frac{d}{dz}\left(\frac{1}{E(z)}\right) - 1,$$

one finds the dependence of deceleration parameter on the redshift in the considered model

$$q(z) = -1 + \frac{3}{2}\Omega_{m0}\frac{(1+z)^{3(1+\beta)}}{E^{(2+3\beta)}} \,. \tag{193}$$

The effective parameter of the state equation is known to equal

$$w_{\text{eff}} \equiv \frac{p_{tot}}{\rho_{tot}} = \frac{(2q-1)}{3}.$$

The figure 4 presents the plots for dependence of some cosmological parameters on the redshift z. The free parameters Ω_{m0} and β were chosen to provide the best agreement with observations. One can find that in the considered model the transition from the decelerated expansion ($q > 0$) to the accelerated one ($q < 0$) took place at $z_t = 0.7489$, the parameter β is negative and therefore dark matter decays into dark energy when $z > z_t$, and vice versa at $z < z_t$. The Universe is interaction-free in dark sector at z_t.

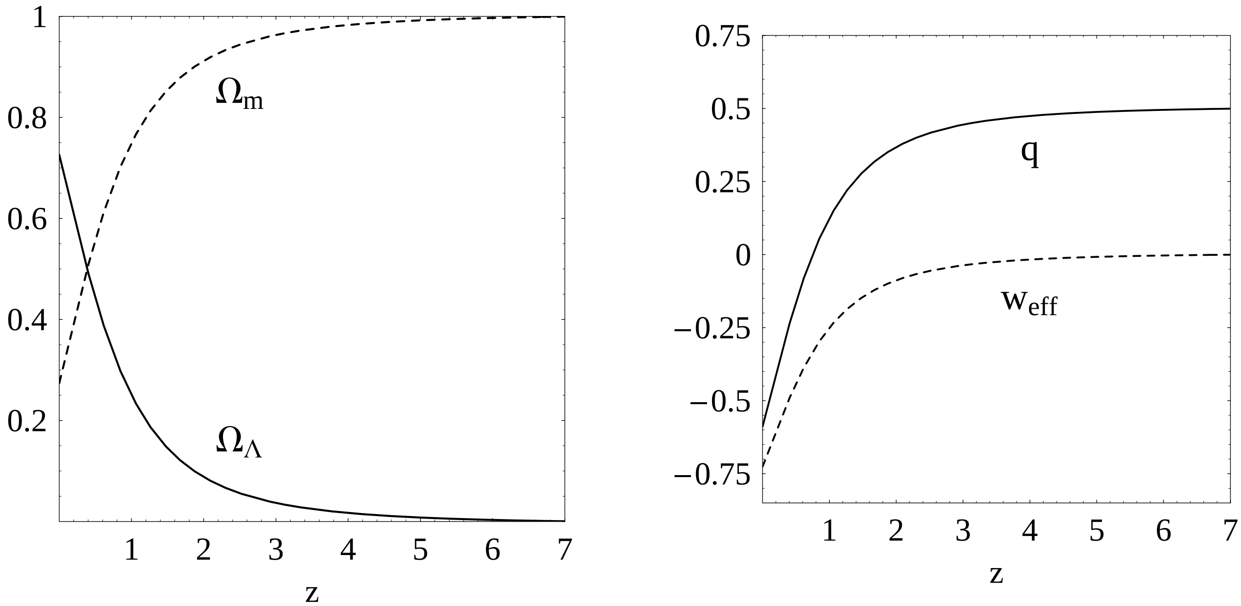

FIG. 4: Ω_m, Ω_Λ, q and $w_{\rm eff}$ as function of the redshift z at $\Omega_{m0} = 0.2738$ and $\beta = -0.010$ in the case $Q = 3\beta q H \rho_m$.

2. Case $Q = q(\alpha\dot{\rho}_{tot} + 3\beta H\rho_{tot})$

Now we consider the case (178), and proceeding completely analogous the above considered case one obtains

$$Q = \frac{3qH^3}{\kappa^2}\left(2\alpha\frac{\dot{H}}{H^2} + 3\beta\right). \tag{194}$$

Inserting the equations (184) and (194) into (144), and transforming to differentiation with respect to the scale factor, one finds

$$aH'' + \frac{a}{H}H'^{\,2} + (4 + 3\alpha q)\,H' + \frac{9\beta q H}{2a} = 0\,. \tag{195}$$

As in the previous case, we obtained again the differential equation of second order for the function $H(a)$. Exact solution exists only in the case $\alpha = 0$:

$$H(a) = C_{22}\cdot a^{-3(2-3\beta+r_1)/8}\cdot\left(a^{3r_1/2} + C_{21}\right)^{1/2}, \tag{196}$$

where C_{21}, C_{22} are integration constants and $r_1 \equiv \sqrt{4 + \beta\,(4 + 9\beta)}$. Inserting (196) into (188), we get

$$\Omega_m = \frac{1}{4}\left[2 - 3\beta + \left(\frac{2C_{21}}{a^{3r_1/2} + C_{21}} - 1\right)r_1\right]. \tag{197}$$

The integration constants are determined as usual from the condition $\Omega_m(a=1) = \Omega_{m0},\ H(a=1) = H_0$:

$$C_{21} = -1 + \frac{2\,r_1}{2 - 3\beta - 4\Omega_{m0} + r_1},\quad C_{22} = H_0\,(1 + C_{21})^{-1/2}\,. \tag{198}$$

We finally get

$$E \equiv \frac{H}{H_0} = (1+z)^{3(2-3\beta+r_1)/8}\cdot\left[\frac{(1+z)^{-3r_1/2} + C_{21}}{1 + C_{21}}\right]^{1/2}. \tag{199}$$

In the considered model there are also two free parameters Ω_{m0} and β. Using the condition $0 \le \Omega_m \le 1$ with $a \to 0$, from the equation (197) one gets $\beta \ge 0$. The best agreement of the model under consideration with observational data

occurs with $\Omega_{m0} = 0.2701$ and $\beta = 0.0$. It means that the considered interaction model gives worse agreement with observations than ΛCDM. More detailed discussion an interested reader finds in the paper [78] by the author of the considered model. The transition form the decelerated expansion phase ($q > 0$) to the accelerated one ($q < 0$) occurs at $z_t = 0.7549$.

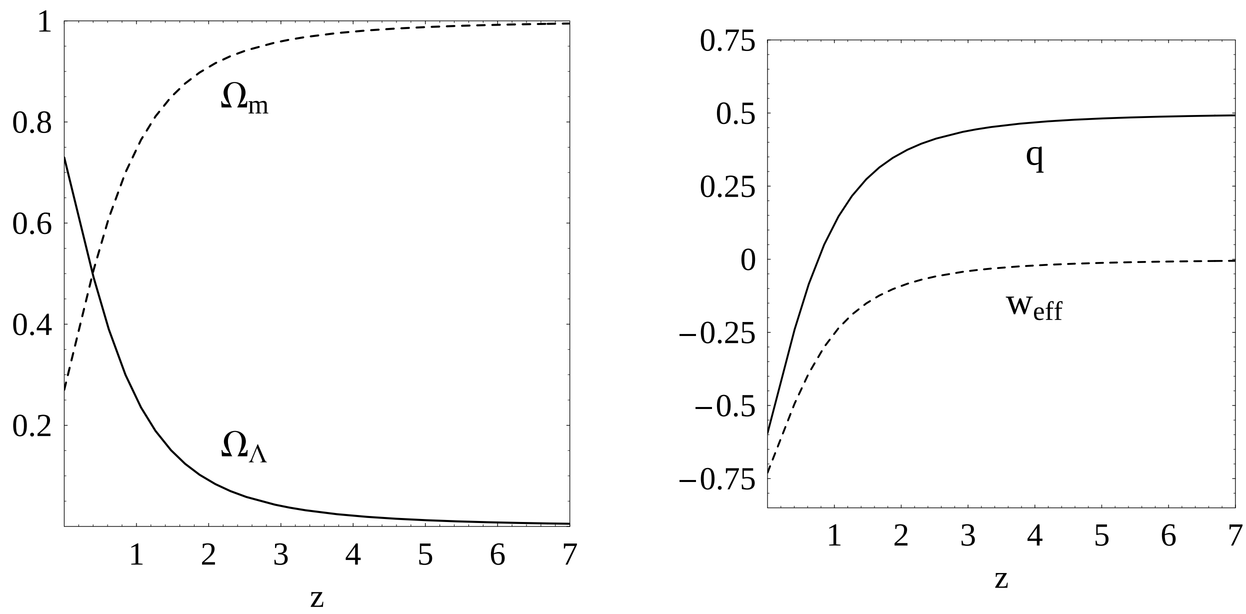

FIG. 5: The same as on Fig. 4, but in the case of interaction of the form $Q = 3\beta q H\rho_{tot}$ under condition $\beta \geq 0$.

3. The case of $Q = q(\alpha\dot{\rho}_\Lambda + 3\beta H\rho_\Lambda)$

For the conclusion we consider the case (179). Following the same procedure as in the two preceding case, one obtains

$$Q = \frac{3\beta q H\rho_\Lambda}{1+\alpha q}\,. \tag{200}$$

With the equation (184) one has

$$\rho_\Lambda = \frac{3}{\kappa^2}H^2 - \rho_m = \frac{1}{\kappa^2}\left(3H^2 + 2\dot{H}\right). \tag{201}$$

Therefore the equation for the Hubble parameter in terms of the scale factor takes the form:

$$aH'' + \frac{a}{H}H'^2 + \left(4 + \frac{3\beta q}{1+\alpha q}\right)H' + \frac{9\beta q H}{2a(1+\alpha q)} = 0. \tag{202}$$

Exact solution can be obtained in the case, $Q = 3\beta q H\rho_\Lambda$, namely

$$H(a) = C_{32}\cdot a^{-3(2-5\beta+r_2)/[4(2-3\beta)]}\cdot\left(a^{3r_2/2} + C_{31}\right)^{1/(2-3\beta)}, \tag{203}$$

where C_{31}, C_{32} are the integration constants, and $r_2 \equiv \sqrt{(2-\beta)^2} = |\,2-\beta\,|$. Inserting (203) into (188), we get

$$\Omega_m = \frac{1}{2\,(2-3\beta)}\left[2 - 5\beta + \left(\frac{2C_{31}}{a^{3r_2/2} + C_{31}} - 1\right)r_2\right]. \tag{204}$$

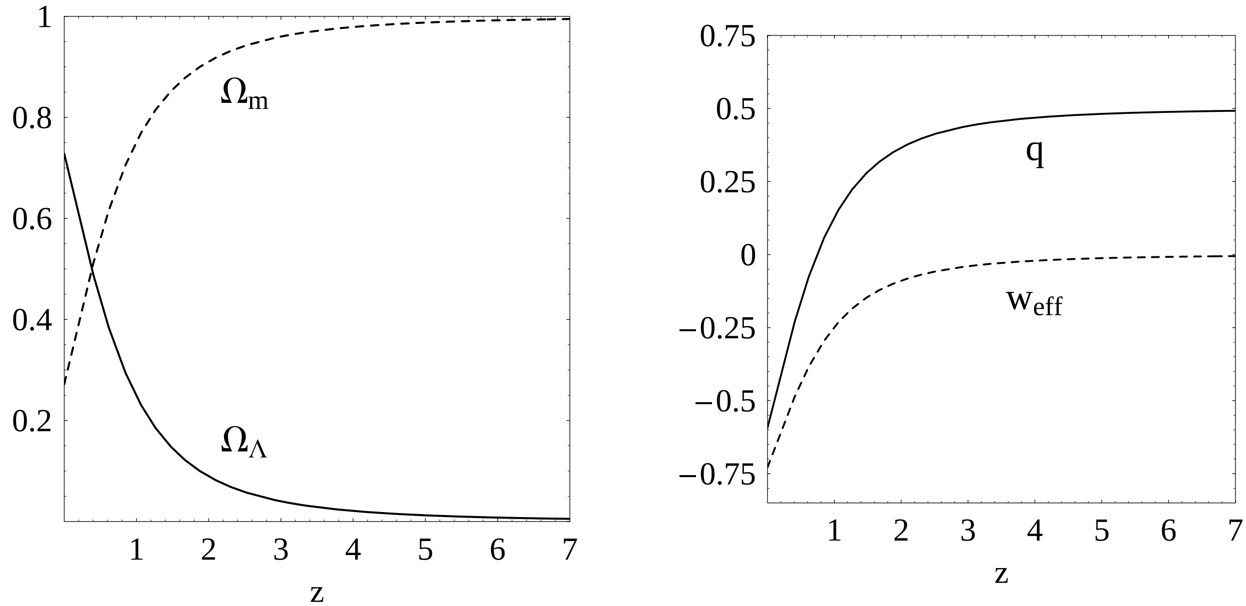

FIG. 6: The same as Fig. 4, but for the case $Q = 3\beta q H \rho_\Lambda$.

Assuming that $\Omega_m(a=1) = \Omega_{m0}$ and $H(a=1) = H_0$, we can write

$$C_{31} = -1 + \frac{2\,r_2}{2 - 5\beta + r_2 + 2\Omega_{m0}\,(3\beta - 2)}, \quad C_{32} = H_0\,(1 + C_{31})^{1/(3\beta-2)}\,, \tag{205}$$

and finally get

$$E \equiv \frac{H}{H_0} = (1+z)^{3(2-5\beta+r_2)/[4(2-3\beta)]} \cdot \left[\frac{(1+z)^{-3r_2/2} + C_{31}}{1 + C_{31}}\right]^{1/(2-3\beta)} . \tag{206}$$

As before, the model has two free parameters Ω_{m0}, β. The maximum plausibility method for the free parameters of the considered model gives the result [78] $\Omega_{m0} = 0.2717,\ \beta = 0.0136$. Unlike the two preceding models, the observational data analyzed in [78], give evidence in favor of $\beta > 0$. More detailed discussion the interested reader can find in the paper [78] of the author of the model.

The plot 6 displays the dependence for deceleration parameter and effective parameter of the state equation $w_{\rm eff} \equiv p_{tot}/\rho_{tot} = (2q-1)/3$ as functions of the redshift z with the parameters obtained by the maximum plausibility method. It is easy to show that the transition from the decelerated expansion phase ($q > 0$) to the accelerated one ($q < 0$) takes place at $z_t = 0.7398$. As the parameter β obtained from observations satisfies $\beta > 0$, then dark energy decays into dark matter ($Q > 0$) for $z > z_t$, and vice versa ($Q < 0$) for $z < z_t$.

D. Statefinders for the interacting dark energy

In this part of the review, we discuss recently introduced "statefinder parameters" [95], that include the third derivative of the cosmic scale factor, are useful tools to characterize interacting quitessence models [128].

Accelerated expansion of Universe is normally described by the deceleration parameter $q = -\ddot{a}/(aH^2)$. As was mentioned above, current value of the Universe deceleration parameter q is negative, however its amplitude is difficult to determine from the observations. Since the accelerated dynamics of Universe is predicted by many models, they can be used to obtain an additional information on q.

In particular, an example of such models is given by the cosmological models where the dominant components – dark matter and dark energy – interact with each other. The models, where the dominating components do not evolve

independently and interact with each other instead, are of special interest because, as was mentioned above, they can facilitate or even completely solve the "coincidence problem".

Let us consider the Universe filled by two components of non-relativistic matter (indexed by m) with negligibly small pressure $p_m \ll \rho_m$ and dark energy (indexed by x) with the state equation $p_x = w\rho_x$, where $w < 0$. The dark energy decays into dark matter according to

$$\begin{aligned} \dot{\rho}_m &+ 3H\rho_m = Q\,, \\ \dot{\rho}_x &+ 3H(1+w)\rho_x = -Q\,, \end{aligned} \tag{207}$$

where $Q \geq 0$ measures the strength of the interaction. For further convenience we will write it as $Q = -3\Pi H$, where Π is a new variable with dimension of pressure.

The Einstein equations for spatially flat Universe have the form

$$H^2 = \frac{8\,\pi G}{3}\rho\,, \tag{208}$$

$$\dot{H} = -\frac{8\,\pi G}{2}(\rho + p_x)\,, \tag{209}$$

where $\rho = \rho_m + \rho_x$ is the total energy density. The quantity $\dot{H}$ is connected by the deceleration parameter q by the relation $q = -1 - (\dot{H}/H^2) = (1 + 3w\Omega_x)/2$, where $\Omega_x \equiv 8\,\pi G\rho_x/(3H^2)$. Evidently the deceleration parameter does not depend on interaction between the components. Nevertheless, differentiation of $\dot{H}$ yields

$$\frac{\ddot{H}}{H^3} = \frac{9}{2}\left(1 + \frac{p_x}{\rho}\right) + \frac{9}{2}\left[w\,(1+w)\,\frac{\rho_x}{\rho} - w\frac{\Pi}{\rho} - \frac{\dot{w}}{3H}\frac{\rho_x}{\rho}\right]\,. \tag{210}$$

In contrast with H and $\dot{H}$, the second order derivative $\ddot{H}$ in fact depends on the interaction of the components. Therefore, in order to distinguish models with different types of interaction, or between interacting and free models, we need the cosmology dynamical description involving the parameters explicitly dependent on $\ddot{H}$. In the papers [96] and [95] Sahni and Alam introduced the pair of parameters (the so-called "statefinders"), which seem to be promising candidates for investigation of distinction between the models with interacting components.

$$r = \frac{\dddot{a}}{aH^3}, \qquad s = \frac{r-1}{3(q-\frac{1}{2})}\,. \tag{211}$$

In the considered context of interacting components they take on the form

$$r = 1 + \frac{9}{2}\frac{w}{1+\kappa}\left[1 + w - \frac{\Pi}{\rho_x} - \frac{\dot{w}}{3wH}\right]\,, \tag{212}$$

where $\kappa \equiv \rho_m/\rho_x$, and

$$s = 1 + w - \frac{\Pi}{\rho_x} - \frac{\dot{w}}{3wH}\,. \tag{213}$$

For the interaction-free models with $\Pi = 0$, the statefinders reduce to the expressions that were studied in [95] and [96]. Let us show how the statefinders can be useful for analysis of cosmological models, where the dominant components interact with each other. Note that the third order derivative of the scale factor is necessary for description of any variations in the general state equation for the cosmic medium [18, 19]. It is obvious from the general relation [96]

$$r - 1 = \frac{9}{2}\frac{\rho + P}{P}\frac{\dot{P}}{\dot{\rho}}\,, \tag{214}$$

where P is the total density of cosmic medium which in our case leads to the relation $P \approx p_x$. Since

$$\left(\frac{d}{dt}\right)\left(\frac{P}{\rho}\right) = \frac{\dot{\rho}}{\rho}\left[\frac{\dot{P}}{\dot{\rho}} - \frac{P}{\rho}\right]\,, \tag{215}$$

it is evident that the interaction term in the relation $\dot{P} \approx \dot{p}_x = \dot{w}\rho_x + w\dot{\rho}_x$ in accordance with (207) will additionally affect the time dependence of the common state parameter P/ρ.

Models with interacting components suggest a dynamical approach to the coincidence problem. The principal quantity in that context is the density ratio κ, which was defined above in expression (212). Strict solution of the coincidence problem requires that the density ratio should be of order of unity on late stages of Universe evolution. At least it has to vary sufficiently slowly for time interval of order H^{-1}. The ratio κ is governed by the equation (207)

$$\dot{\kappa} = -3H\left[\left(\frac{\rho_x+\rho_m}{\rho_m\,\rho_x}\right)\Pi - w\right]\kappa\,. \tag{216}$$

Now let us consider the statefinders for different solutions of that equation.

E. Scaling solutions

In the paper [98] it was shown by the authors that the so-called scaling solutions, i.e. the solutions of the form $\rho_m/\rho_x \propto a^{-\xi}$, where ξ is constant parameter, lying in the range $[0,3]$, can be calculated in the case when when dark energy decays into dark matter (see eq.(207)). The solutions of such type make interest because they facilitate solution of the coincidence problem [99]. Indeed, the model with $\xi = 3$ reduces to ΛCDM one for $w = -1$ and $\Pi = 0$. For $\xi = 0$ the Universe dynamics becomes stable with $\kappa = const$, and the coincidence problem does not appear [100]. Thus for the parameter values $\xi < 3$ the coincidence problem weakens. In this scheme with $w = const$, one can easily see that the interaction which generates the scaling solutions, can be calculated as the following

$$\frac{\Pi}{\rho_x} = \left(w+\frac{\xi}{3}\right)\frac{\kappa_0(1+z)^\xi}{1+\kappa_0(1+z)^\xi}\,, \tag{217}$$

where κ_0 denotes the current ratio of the energy densities and $z = (a_0/a) - 1$ is the redshift. Substituting this expression into the equations (212) and (213), one finds the statefinder parameters:

$$r = 1 + \frac{9}{2}\frac{w}{1+\kappa_0(1+z)^\xi}\left[1 + w - \left(w+\frac{\xi}{3}\right)\frac{\kappa_0(1+z)^\xi}{1+\kappa_0(1+z)^\xi}\right]\,, \tag{218}$$

and

$$s = 1 + w - \left(w+\frac{\xi}{3}\right)\frac{\kappa_0(1+z)^\xi}{1+\kappa_0(1+z)^\xi}. \tag{219}$$

Figure 7 presents the dependence $r(s)$ for different values of ξ. The less is the value ξ, the lower is the corresponding curve on s–r diagram and the less actual is the coincidence problem. Remark that those curves are qualitatively identical to those drawn for the models with interaction-free components (see Fig.8 [95]). Compare also that ΛCDM model ($\Pi = 0$, $w = -1$) corresponds to the point $s = 0$, $r = 1$ (not shown on the figure). The present values r_0 and s_0 of the statefinder parameters are important both from the observational point of view and for the purpose of distinction between different models. For the scaling models one has

$$r_0 = 1 + \frac{9}{2}\frac{w}{1+\kappa_0}s_0\,, \qquad \text{and} \qquad s_0 = 1 + w - \left(w+\frac{\xi}{3}\right)\frac{\kappa_0}{1+\kappa_0}\,. \tag{220}$$

Having in mind that

$$q_0 = \frac{1}{2}\frac{1+\kappa_0+3w}{1+\kappa_0}\,, \tag{221}$$

and introducing

$$q_{0\Lambda} \equiv q_0\,(w=-1) = -\frac{1}{2}\frac{2-\kappa_0}{1+\kappa_0} \Leftrightarrow \frac{3}{2}\frac{\kappa_0}{1+\kappa_0} = 1 + q_{0\Lambda}\,, \tag{222}$$

we can classify different models by its dependence $s_0(q_0)$, which is

$$s_0 = \frac{2}{3}\left[q_0 - q_{0\Lambda} + \left(\frac{\xi}{3}-1\right)(1+q_{0\Lambda})\right]\,. \tag{223}$$

The first part of the bracket in the righthand side describes deviation from $w = -1$, and the second responds for deviation from ΛCDM value $\xi = 3$. For models with $w = -1$, which have the same deceleration parameter $q_0 = q_{0\Lambda}$, we obtain $s_0 = \frac{2}{3}\left(\frac{\xi}{3}-1\right)(1+q_{0\Lambda})$. Of course, $\xi = 3$ corresponds to ΛCDM model with $s_0 = 0$. If the conditions $\kappa_0 = 3/7$ and $\xi = 1$ hold, then $s_0\,(\xi = 1) = -0.2$, while for $\xi = 0$ one has $s_0\,(\xi = 0) = -0.3$.

Analogous considerations are valid also for other values of w. Therefore the parameter s_0 enable us to distinguish between different scaling models, which have the same deceleration parameter.

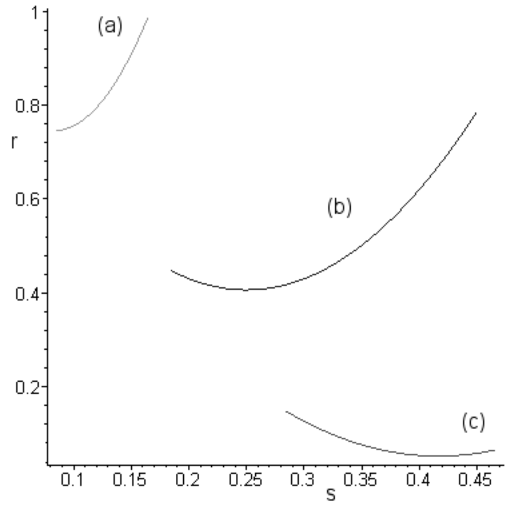

FIG. 7: The curves $r(s)$ are drawn for the redshift range $[0, 6]$ (from left to right) with $w = -0.95$ and $\kappa_0 = 3/7$ for 3 different values of the ξ parameter: (a) 2.5; (b) 1.5; (c) 0.5.

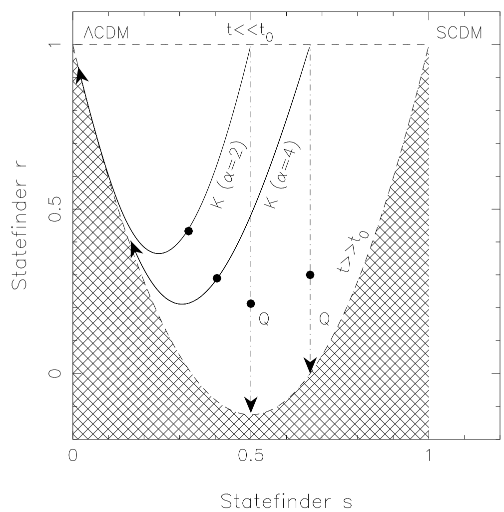

FIG. 8: The Statefinder pair (r, s) is shown for different forms of dark energy. In quintessence (Q) models (w =constant $\neq -1$) the value of s remains fixed at $s = 1 + w$ while the value of r asymptotically declines to $r(t \gg t_0) \simeq 1 + \frac{9w}{2}(1 + w)$. Two models of quintessence corresponding to $w_Q = -0.25, -0.5$ are shown. Quintessence (K) models are presented by a scalar field (quintessence) rolling down the potential $V(\phi) \propto \phi^{-\alpha}$ with $\alpha = 2, 4$. ΛCDM ($r = 1, s = 0$) and SCDM in the absence of l ($r = 1, s = 1$) are the fixed points of the system. The hatched region is disallowed in quintessence models and in the quintessence model which we consider. The filled circles show the *current values* of the statefinder pair (r, s) for the Q and K models with $\Omega_{m0} = 0.3$.

F. Concluding remarks

The statefinder parameters introduced in [95] and [96], as was assumed, became useful tools for verification of models with interacting components, which in turn solve or weaken the coincidence problem — a stone of obstacle for many models with late acceleration. While the deceleration parameter does not depend on interaction between the dark energy and dark matter, the state finders (r, s) contain the dependence explicitly.

IX. HOLOGRAPHIC DYNAMICS: ENTROPIC ACCELERATION

In the previous section we presented the description of Universe dynamics based on general relativity with some generalizations. The present section is intended to show that there exists a principally independent approach, which enables us both to reproduce all achievements of the traditional description and resolve a number of problems the latter faced to.

The traditional point of view assumed that space filling fields constitute the dominant part of degrees of freedom in our World. However it became clear soon that such estimate much hardened the development of the quantum gravity theory: for the latter to make sense one had to cutoff on small distances all the integrals appeared in the theory. Consequently our World was described on a three-dimensional discrete lattice with cell size of order of Planck length.

Recently some physicists came up with even more radical point of view: complete description of Nature required only two-dimensional lattice, situated on space edge of our World, instead of the three-dimensional one. Such approach is based on so-called "holographic principle" [108, 109, 113, 116, 141–143]. The term comes from the optical holography, which represent nothing but two-dimensional recording of three-dimensional objects. The holographic principle is composed of the two main statements:

1. all information contained in some region of space can be "recorded" (presented) on boundary of that region;
2. the theory contains at most one degree of freedom per Planck area on boundaries of the considered space region

$$N \leq \frac{Ac^3}{G\hbar}. \tag{224}$$

Therefore central place in the holographic principle is occupied by the assumption that all information about the Universe can be coded on some two-dimensional surface — the holographic screen. Such approach leads to possibly new interpretation of cosmological acceleration and to completely novel concept of gravity. The gravitation is now defined as entropic force generated by variation of information connected to positions of material bodies.

Let us consider a small piece of holographic screen and approaching particle of mass m. According to the holographic principle, the particle affects the information amount (and thus on entropy), which is stored on the screen. It is naturally to assume that the entropy increment near the screen is linear on the displacement Δx,

$$\Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x. \tag{225}$$

The factor 2π is introduced for convenience reasons, to be clear below. To make clear why that quantity is proportional to mass, let us imagine that the particle splits on two or more particles of lower mass. Each of them then carries its own entropy increment with displacement on Δx. As both entropy and mass are additive function, then they are proportional to each other. According to the first principle of thermodynamics, the entropic force, caused by information variation, reads

$$F\Delta x = T\Delta S. \tag{226}$$

If the entropy gradient is given, or calculated using the relation (225), and the screen temperature is fixed, then one can find the entropy force. As is known, an observer, moving with acceleration a, effectively feels the so-called Unruh temperature [144]

$$k_B T = \frac{1}{2\pi}\frac{\hbar}{c}a. \tag{227}$$

Let us assume that the total energy of the system equals E. A simple assumption follows, that it is uniformly distributed on all N bits of information on the holographic screen. The temperature is then defined as average energy per bit

$$E = \frac{1}{2}Nk_B T. \tag{228}$$

The formulae (224)-(228) enable us to formulate the holographic dynamics, and in particular dynamics of Universe, without any notion of gravity.

The above presented ideology is essentially based on successful description of black holes physics [151–154]. On the first face, there is nothing common between the extremely diluted Universe with density $\rho \sim 10^{-29} g/cm^3$ and "typical" black hole of stellar mass with critical density $\rho \sim 10^{14} g/cm^3\,(M = 10M_\odot)$. But the situation sharply

changes if we transit to more massive black holes. As we have seen above, the black hole radius scales $r_g \propto M$, therefore for the black hole critical density one has $\rho \propto M^{-2}$. Super-massive black holes situated in the galactic nuclei reach masses of order $10^{10} M_\odot$, and their gravitational radius ($r_g \sim 3\times 10^{15} cm$) five times exceeds the dimensions of Solar system. The critical density in the latter case equals $\rho \sim 10^{-4} g/cm^3$, which is one order of magnitude less than the room air density ($1.3\times 10^{-3} g/cm^3$). Let us consider, how close are the parameters of our observed Universe to those of black hole. We estimate mass of the observed Universe taking the Hubble radius for its radial dimension H^{-1} The mass contained inside the Hubble sphere thus reads

$$M_{univ} = \frac{4\pi}{3} R_H^3 \rho. \tag{229}$$

In the case of flat space [4], the density ρ of the observable part of Universe enclosed by Hubble sphere can be reasonably replaced by the critical one (16) $\rho = 3H^2/(8\pi G)$, and therefore the formula (229) takes the form

$$M_{univ} = \frac{R_H}{2G}.$$

Inserting the obtained mass value into the expression for gravitational radius of the Universe, $r_g = 2GM_{univ}$, one finds

$$r_g = R_H. \tag{230}$$

The obtained result for many reasons is nothing but a rough estimate, however it obviously favors the above cited black hole argumentation in application to the observable Universe. Remark that in the case of the Sun the ratio of the physical radius $R_\odot = 695\,500 km$ to the gravitational one $r_{g\odot} = 3km$ counts more than five orders of magnitude.

Now let us identify the holographic screen with the Hubble sphere of radius $R = H^{-1}$ (which is valid for flat Universe) and we try, making use of the holographic principle, to reproduce the Friedmann equations, using neither Einstein equations nor Newtonian mechanics. The holographic screen spans the area $A = 4\pi R^2$ and carries the (maximum) information $N = 4\pi R^2/L_{Pl}^2$ bits. The change of information quantity dN for the time period dt, caused by the Universe expansion $R \to R + dR$, equals

$$dN = \frac{dA}{L_{Pl}^2} = \frac{8\pi R}{L_{Pl}^2} dR.$$

Here we use the system of units $c = k_B = 1$. Variation of Hubble radius leads to change in Hawking temperature $\left(T = \frac{\hbar}{2\pi R}\right)$

$$dT = -\frac{\hbar}{2\pi R^2} dR.$$

From the equipartition law it follows that

$$dE = \frac{1}{2} N dT + \frac{1}{2} T dN = \frac{\hbar}{L_{Pl}^2} dR = \frac{dR}{G}, \tag{231}$$

$\left(L_{Pl}^2 = \frac{\hbar G}{c^3}\right)$. The quantity dR can be presented in the form

$$dR = -H\dot{H} R^3 dt. \tag{232}$$

From the other hand, the energy flow through the Hubble sphere can be calculated if the energy-momentum tensor is given for the substance filling the Universe. Considering the latter substance as an ideal liquid, (see chapter 2) and using the relation $T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$, we obtain

$$dE = A\,(\rho + p)\,dt. \tag{233}$$

Equating (231) to (233) accounting (234), one finds

$$\dot{H} = -4\pi G\,(\rho + p)\,.$$

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[4] as we have repeatedly mentioned above, deviations from the flatness are very small

It is well known that the system of equations

$$\dot{H} = -4\pi G\,(\rho + p)\,,\ \ \dot{\rho} + 3H\,(\rho + p) = 0. \tag{234}$$

is equivalent to the system (13)-(14). Thus the declared aim is achieved. We remark that already in the year 1995 a more general problem was solved [119]: the Einstein equations were obtained from the thermodynamic considerations. This important result followed from the fact that entropy was proportional to horizon area and from the assumption that the relation (227) holds for any accelerated moving observer situated inside her own casual horizon, and T stands for Unruh temperature.

Derivation of the Friedmann equations from the holographic principle is of course an important result, but by itself it represents nothing but reproduction of something already well-known. A natural question is whether one can develop a novel approach to description of Universe dynamic basing on the holographic principle? If yes, than is it possible to overcome the unsolvable difficulties of the traditional approach in frames of the new one? Let us start from the logic scheme which the holographic dynamics of Universe is based on.

The standard procedure to develop any field theory involves the formulation of equations of motion obtained from some effective action corresponding to the energy scale characteristic to the phenomenon to be described by the theory. General relativity theory is not an exception from the rule: equations of motion in general relativity are obtained by variation of effective action with respect to dynamical variables present in it. The standard approach to evaluate the integrals that arise in variations of the action is the integration by parts, which produces the terms corresponding to boundaries of the region under consideration. According to such approach a quite reasonable assumption is made that on boundary of the considered region, far from the field sources, all the dynamical variables, be it either fields, or gravitational potentials or connectivity components (as in general relativity), turn to zero. In holographic physics instead the surface is the place where the main action takes goes on. The natural conclusion follows that the contribution of surface terms now should be necessarily accounted.

Let us show that consideration of the boundary term in the Einstein-Hilbert action is equivalent to inclusion of non-zero energy-momentum tensor in the standard Einstein equations. The action for gravitational field takes the form †

$$S_{EH} \;=\; -\frac{c^3}{16\pi G}\int d^4x\,\sqrt{-g}\,R\,. \tag{235}$$

Variation of that action with respect to metrics $g_{\mu\nu}$ in a compact region Ω, one obtains

$$\begin{aligned}&\delta\int_\Omega d^4x\sqrt{-g}\,R\;=\\ &\quad\int_\Omega d^4x\sqrt{-g}\Bigg[\Big(g^{\mu\nu}\nabla^2\delta g_{\mu\nu}\;-\nabla^\mu\nabla^\nu\delta g_{\mu\nu}\Big)-\bigg(R^{\mu\nu}-\frac{1}{2}\,g^{\mu\nu}R\bigg)\,\delta g_{\mu\nu}\Bigg]\,,\end{aligned} \tag{236}$$

where $\nabla^2\;=\;g^{\alpha\beta}\nabla_\alpha\,\nabla_\beta$. Full derivative in (235) can be presented as the contribution of the region boundary $\partial\,\Omega$, where those contribution do not cancel. In order to obtain the Einstein equations from the least action principle without boundary terms, the Einstein-Hilbert action should be complemented by a functional intended to compensate the contribution of full derivative in (235). We denote it as $S_{Boundary}[g]$.. Then the full action takes on the form

$$S\;=\;-\frac{1}{16\pi G}\int_\Omega d^4x\sqrt{-g}\,R+\;S_{Boundary}[g]\;+\;S_{Source}\,, \tag{237}$$

where we included possible sources S_{Source} of gravitational field, coupled with the material fields. The functional $S_{Boundary}[g]$, thus takes the form

$$\delta S_{Boundary}[g]\;=\;-\,\frac{1}{16\pi G}\int_\Omega d^4x\sqrt{-g}\Big(g^{\mu\nu}\nabla^2\delta g_{\mu\nu}\;-\;\nabla^\mu\nabla^\nu\delta g_{\mu\nu}\Big)\,, \tag{238}$$

In the context of holographic physics where the boundaries play crucial role, the functional $S_{Boundary}[g]$ can be treated as action for holographic dark energy. The Einstein equations including the contributions of material fields

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† We use the signature $(+,-,-,-)$, and definition $R^\rho{}_{\sigma\mu\nu}=\partial_\mu\Gamma^\rho_{\nu\sigma}-\partial_\nu\Gamma^\rho_{\mu\sigma}+\Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma}-\Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}$, $R_{\nu\mu}=R_{\nu\beta\mu}{}^\beta$, $R=g^{\mu\nu}R_{\mu\nu}$.

and boundary terms have the following form

$$R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R \;=\; 8\pi G\left(T^{\mu\nu}_{Source}+T^{\mu\nu}_{Boundary}\right),$$
$$T^{\mu\nu}_{Source} \;=\; \frac{2}{\sqrt{-g}}\frac{\delta S_{Source}}{\delta g_{\mu\nu}},\; T^{\mu\nu}_{Boundary}=\frac{2}{\sqrt{-g}}\frac{\delta S_{Boundary}}{\delta g_{\mu\nu}}. \tag{239}$$

Remark that in the flat space case, when $g_{\mu\nu}=0$, the boundary action $S_{Boundary}[g]=0$, and Einstein equations take the standard form. We also note that such modification of Einstein equations preserves the general structure of the equations, which, as we will see below, guarantees the conservation of the Friedmann equation structure.

Inclusion of the additional terms in the Einstein equations requires the so-called "holographic correction" of the Friedmann equations. The easiest way to perform the correction is to insert the entropic force into the second Friedmann equation [120]. The entropic term structure can be guessed from the dimensional considerations

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3p)+\frac{a_e}{L_b}. \tag{240}$$

Here $a_e{=}F_e/m$ is the acceleration caused by the entropic force and L_b is the scale length, determined by the holographic screen position. If one takes the Hubble sphere as the holographic screen, then the scale length coincides with the Hubble radius: $L_b=R_H=H^{-1}$. The entropic acceleration magnitude can be expressed through the holographic screen temperature T_b

$$a_e=\frac{F_e}{m}=T_b\frac{\Delta S}{\Delta x}\frac{1}{m}=2\pi T_b. \tag{241}$$

Does the considered scheme agree with the main observational data at least? We have shown above in frames of SCM that absolute value of the cosmological acceleration of the Hubble sphere equals $\dot{V}\simeq 4\times 10^{-10}m\mathrm{sec}^{-2}$. Let us estimate that magnitude in the holographic approach.

It is naturally to assign the Unruh temperature to the holographic screen $T_b=T_U$. Let us link it with the Hawking radiation temperature T_H, which is equal to

$$T_H=\frac{\hbar}{8\pi k_B GM}=\frac{\hbar g}{2\pi k_B}, \tag{242}$$

where g is the free fall acceleration on the Hubble sphere. Comparing the latter expression with Unruh temperature

$$T_U=\frac{\hbar a}{2\pi k_B}, \tag{243}$$

one gets

$$T_H=T_U(a=g). \tag{244}$$

Thus the Unruh temperature coincides with that for the Hawking radiation, but depends on the reference frame acceleration instead of surface gravitation (free fall acceleration). According to the equivalence principle, the free fall acceleration on the Hubble surface is equivalent to acceleration of a reference frame and we can relate it to each other. Treating the Hubble sphere as an analogue of events horizon for a black hole, we can present the formula (242) in the following form

$$T_H=\frac{\hbar}{k_B}\frac{H}{2\pi}\sim 3\times 10^{-30}\,K. \tag{245}$$

Comparing (245) and (243), one finds that the Hubble sphere acceleration a_H equals

$$a_H=H\simeq 10^{-9}m\,\mathrm{sec}^{-2}. \tag{246}$$

The latter result is in convincing qualitative agreement with the one obtained in SCM.

Certainly, quality of a novel model is not defined by its ability to reproduce the results established by its predecessors — it is nothing but minimum task. Its main purpose and virtue is to solve the problems untouchable by former models. The holographic dynamics is very promising from that point of view as it gives hope to solve both main challenges of the SCM — they are cosmological constant magnitude and coincidence problem.

Let us start from the former. The cosmological constant problem stands for the huge disagreement [6] between the observed density of dark energy in form of the cosmological constant and its "expected" value. The expectations are based on rather natural assumptions on the cutoff parameter for the integral, representing the zero fluctuations of vacuum. The holographic principle enables us to replace the "natural assumptions" by more thorough quantitative estimates.

A. Universe with holographic dark energy

In any effective quantum field theory defined in space region with typical length scale L and using the ultraviolet cutoff Λ, the system entropy takes the form $S \propto \Lambda^3 L^3$. For instance, fermions, placed in nodes of space lattice with characteristic size L and period Λ^{-1}, occupy one of the number $2^{(L\Lambda)^3}$ states. Therefore entropy of such system is $S \propto \Lambda^3 L^3$. According to the holographic principle, this quantity must obey the inequality [121]

$$L^3\Lambda^3 \le S_{BH} \equiv \frac{1}{4}\frac{A_{BH}}{l_{Pl}^2} = \pi L^2 M_{Pl}^2, \tag{247}$$

where S_{BH} is the black hole entropy and A_{BH} stands for its event horizon area, which in the simplest case coincides with the surface of sphere with radius L. This reasoning shows that magnitude of the infrared cutoff cannot be chosen independently of the ultraviolet one.

Thus we formulate the important result [121]: in frames of holographic dynamics the infrared cutoff magnitude is strictly linked to the ultraviolet one. In other words, physical properties on small UV-scales depends on physical parameters on large IR-scales. In particular, if the inequality (247) holds, then one gets

$$L \sim \Lambda^{-3} M_{Pl}^2. \tag{248}$$

Effective field theories with UV-cutoff (248) obviously include many states with the gravitational radius exceeding the region where the theory was initially defined. In other words, for arbitrary cutoff parameter one can find sufficiently large volume where entropy in an effective field theory exceeds the Bekenstein limit. In order check it let us remark that usually an effective quantum field theory must be able to describe the system under temperature $T \le \Lambda$. While $T \gg 1/L$, the system has thermal energy $M \sim L^3T^4$ and entropy $M \sim L^3T^3$. The condition (247) is satisfied at $T \le \left(M_{Pl}^2/L\right)^{1/3}$, which responds to gravitational radius $r_g \sim L(LM_{Pl}) \gg L$. In order to get rid of that difficulty an even more strict limitation [121] is imposed on the infrared cutoff $L \sim \Lambda^{-1}$, which excludes all the states localized within limits of their gravitational radii. Taking into account all above mentioned and using the expression (249)

$$\rho_{vac} \approx \frac{\Lambda^4}{16\pi^2}, \tag{249}$$

the condition (247) can be presented in the form

$$L^3\rho_\Lambda \le L M_{Pl}^2 \equiv 2M_{BH}, \tag{250}$$

where M_{BH} is the black hole mass with gravitational radius L. Thus total energy contained in the region of dimensions L cannot exceed the mass of a black hole of roughly the same size. This result agrees surprisingly well with the expression (230) provided the IR-cutoff scale is identified with the Hubble radius H^{-1}, besides that such choice is well motivated in cosmology context. The quantity ρ_Λ is commonly called the holographic dark energy.

In the cosmological aspect under interest the link between the small and large scales can be obtained from rather natural condition, that total energy contained within a volume of dimension L cannot exceed mass of a black hole of the same size:

$$L^3\rho_\Lambda \le M_{BH} \sim L M_{Pl}^2. \tag{251}$$

Here ρ_Λ is the energy density. Would this inequality be violated, the Universe was composed of black holes solely. Applying this relation to the Universe as whole it is naturally to identify the IR-scale with the Hubble radius H^{-1}. Then for the upper bound of the energy density one finds

$$\rho_\Lambda \sim L^{-2} M_{Pl}^2 \sim H^2 M_{Pl}^2. \tag{252}$$

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[6] about 120 orders of magnitude !!!

The quantity ρ_Λ is usually called the holographic dark energy. We will below denote its density as ρ_{DE}. Accounting that

$$M_{Pl} \simeq 1.2 \times 10^{19} GeV; \quad H_0 \simeq 1.6 \times 10^{-42} GeV,$$

one finds

$$\rho_{DE} \sim 10^{-46}\, GeV^4. \tag{253}$$

This expression will agrees with the observed value of the dark energy density $\rho_{DE} \simeq 3 \times 10^{-47} GeV^4$. Therefore the holographic dynamics is free from the cosmological constant problem.

Though the obtained dark energy density corresponds to the correct one, the choice of the IR-cutoff scale equal to the Hubble radius a problem arises connected with the state equation problem: the holographic dark energy cannot provide the accelerated expansion in that case [158]. It can be easily seen from the following simple arguments. Consider the Universe composed of holographic dark energy with density given by the relation (252) and of normal matter component. In that case $\rho = \rho_\Lambda + \rho_M$. From the first Friedmann equation it follows that $\rho \propto H^2$. If $\rho_\Lambda \propto H^2$, then dynamical behavior of holographic dark energy coincides with that of normal matter, thus the accelerated expansion is impossible. If the Universe was initially dominated by matter, then the dark energy defined by (252) is in fact a tracker solution, as it reproduces dynamics of the dominating component. The presence of tracker solutions in frames of the holographic models gives a hope to solve the coincidence problem, but it contradicts to the observed dynamics of Universe.

In order to produce the accelerated expansion of Universe in frames of holographic dark energy model we will try to use the IR-cutoff spatial scale different from the Hubble radius. First which comes in mind is to replace the Hubble radius by the particle horizon R_p:

$$R_p = a\int_0^t \frac{dt}{a} = a\int_0^a \frac{da}{Ha^2}. \tag{254}$$

Unfortunately such substitution cannot give the desired result. It can be made clear again using the initially matter-dominated Universe model. Due to the causality principle the gravity influence cannot extend to the regions situated on distances exceeding the causality horizon size. Therefore the vacuum energy density present in the Friedmann equations cannot be an arbitrary function of L. In the matter dominated era the causality (particle) horizon scales as $a^{3/2}$, therefore $\rho_{_{DE}}(L) \sim a^{-3}$. As $\rho(t) \sim a(t)^{-3(1+w)}$, then it immediately follows that for the case of holographic energy with IR-cutoff on the particle horizon one gets $w = 0$, which again clearly contradicts the observations.

Let us convince ourself now that the above discovered problem preserves also in the case of Universe dominated by holographic dark energy. Present the density of the latter in the following form [158]

$$\rho_{DE} = 3c^2 M_{Pl}^2 L^{-2}. \tag{255}$$

The coefficient $3c^2$ $(c > 0)$ is introduced for convenience, and M_{Pl} further stands for reduced Planck mass: $M_{Pl}^{-2} = 8\pi G$. Replacing $L \to R_h$ in (255), one obtains the first Friedmann equation in the form

$$R_p H = c, \tag{256}$$

and it immediately follows that

$$\frac{1}{Ha^2} = c\frac{d}{da}\left(\frac{1}{Ha}\right). \tag{257}$$

One now easily finds that

$$H^{-1} = \alpha a^{1+\frac{1}{c}}, \tag{258}$$

where α is constant. As $R_h = cH^{-1}$, and $\rho_\Lambda = 3c^2 M_{Pl}^2 R_h^{-2}$, then

$$\rho_{DE} = 3\alpha^2 M_{Pl}^2 a^{-2\left(1+\frac{1}{c}\right)}. \tag{259}$$

How can we find the parameter w in the state equation if the dependence $\rho = f(a)$ is given? One possible approach is the following. Let us use the conservation equation

$$\dot\rho + 3H\rho(1+w) = 0.$$

In the case $\rho = f(a)$ it takes the form

$$f'(a)\dot{a} + 3Hf(a)(1+w) = 0,$$

and it follows that

$$w = -\frac{1}{3}\frac{f'(a)}{f(a)}a - 1 = -\frac{1}{3}\frac{d\ln f(a)}{da}a - 1. \tag{260}$$

It is easy to see that for well-known cases of matter $\rho \propto a^{-3}$, radiation $\rho \propto a^{-4}$ and cosmological constant $\rho = const$ one obtains the correct values $w = 0, 1/3, -1$. From the expression (260) it follows that

$$w = -\frac{1}{3} + \frac{2}{3c} > -\frac{1}{3}. \tag{261}$$

Thus one can see that the above described component does not deserves the name of "dark energy" in proper sense, as it cannot serve its main purpose — to provide the accelerated expansion of universe. The difficulties originate from the fact that, as it follows from (257), the derivative obeys $\frac{d}{da}(H^{-1}/a) > 0$. In order to obtain the accelerated expansion of Universe, one should "slow down" the growth of the IR-cutoff scale. It turns out [123], that it can be done if one replaces the particle horizon by the events one. Recall (see Chapter 2), that the size of spatial region such that a signal emitted at time t from any point of it will reach the immobile observer at infinitely far future, equals

$$R_e = a(t)\int_t^\infty \frac{dt'}{a(t')}. \tag{262}$$

Assuming again that the dark energy dominates and finding solution of the first Friedmann equation in the form

$$\int_a^\infty \frac{da}{Ha^2} = \frac{c}{Ha}, \tag{263}$$

one finds

$$\rho_{DE} = 3c^2M_{Pl}^2R_e^{-2} = 3\alpha^2M_{Pl}^2a^{-2\left(1-\frac{1}{c}\right)}, \tag{264}$$

or $w = -\frac{1}{3} - \frac{2}{3c} < -\frac{1}{3}$. We obtained a component which behaves as the dark energy, i.e. it provides the accelerated expansion of Universe. If $c = 1$ then it behaves as the cosmological constant. If $c < 1$, then $w < -1$. Such value of w responds to phantom model in the traditional approach.

In the above considered cases we investigated properties of holographic dark energy in two limiting cases: the matter dominated and dark energy dominated one. Let us now consider the common case situation, i.e. the Universe dynamics at arbitrary relation between the both components densities [124]. For simplicity we restrict to the case of flat Universe and set the IR-cutoff scale to the event horizon R_e (35). Introducing the relative density of the holographic dark energy $\Omega_{DE} = \rho_{DE}/\rho_{cr}$ ($\rho_{cr} = 3M_{Pl}^2H^2$), we represent (255) for $L = R_e$ in the form

$$HR_e = \frac{c}{\sqrt{\Omega_{DE}}}. \tag{265}$$

Of course, with $\Omega_{DE} = 1$ and replacement $R_e \to R_h$ the equation (265) transits into (256). Taking derivative with respect to time from both sides of (262), one gets

$$\dot{R}_e = HR_e - 1 = \frac{c}{\sqrt{\Omega_D E}} - 1 \tag{266}$$

From the definition (255) it follows that

$$\frac{d\rho_{DE}}{dt} = -6c^2M_{Pl}^2R_e^{-3}\dot{R}_e = -2H\left(1 - \frac{\sqrt{\Omega_{DE}}}{c}\right)\rho_{DE}. \tag{267}$$

Due to the energy conservation law

$$\frac{d}{da}\left(a^3\rho_{DE}\right) = -3a^2p_{DE}. \tag{268}$$

and it follows that

$$p_{DE} = -\frac{1}{3}\frac{d\rho_{DE}}{d\ln a} - \rho_{DE}. \tag{269}$$

Thus the state equation reads

$$w_{DE} = \frac{p_{DE}}{\rho_{DE}} = -\frac{1}{3}\frac{d\ln\rho_{DE}}{d\ln a} - 1 = -\frac{1}{3}\left(1 + \frac{2}{c}\sqrt{\Omega_{DE}}\right). \tag{270}$$

We used the fact that $d\ln a = Hdt$. This result can be obtained without calculation of the pressure p_Λ by the relation (260). The obtained expression for w_{DE} is a consequence of the holographic dark energy definition in form of (255), therefore it is independent on other energy components. It follows from the obtained result that $w_{DE} \simeq -1/3$ in the case of domination of other energy components and $w_{DE} = -\frac{1}{3}\left(1+\frac{2}{c}\right)$ in the dark energy dominated case. The latter result coincides with the expression (260), obtained above for the Universe filled by solely holographic dark energy.

For the first impression the declared task is completed. The holographic dark energy with density (255) from the one hand provides correspondence between the observed density and the theoretical estimate, and from the other it leads to the state equation which is able to generate the accelerated expansion of Universe. However the holographic dark energy with IR-cutoff on the event horizon still leaves unsolved problems connected with the causality principle: according to the definition of the event horizon the holographic dark energy dynamics depends on future evolution of the scale factor. Such dependence is hard to agree with the causality principle.

In order to seek the way out from that dead end let us address once more the cosmological constant problem, which is the huge gap between the theoretical estimates and observed values of the dark energy density. The simplest type of the dark energy — the cosmological constant — is attribute to vacuum average of quantized fields and can be measured in gravitational experiments. Therefore the cosmological constant problem is the quantum gravity one. Although the complete quantum gravity theory is absent, a combination of quantum mechanics with general relativity can shed light on the question under discussion.

From the first days of quantum mechanics the concept of measurements, either real or gedanken, played fundamental role for understanding of physical reality. General relativity states that classical physical laws can be verified with arbitrary unlimited precision. The above revealed connection between the macroscopic (infrared) and microscopic scales dictates the necessity of deeper analysis of the measurement process. The uncertainty relations combined with general relativity generates a fundamental space-time scale — the planck length $L_{Pl} \sim 10^{-33}\,cm$.

The existence of a fundamental length scale critically influence the measurement process [127]. Assume that the fundamental length scale is L_f. As the space-time reference frame must make sense, it must be linked to physical bodies. Therefore the fundamental length postulate is equivalent to limitations for possibilities to realize the precise reference frames. In terms of light signal experiments this means, for instance, that the time interval required for the light signal to propagate from A to b and back, measured by clock in A, is subject to uncontrolled fluctuations. The latter should be treated as evidence for the fluctuations of metric, i.e. of the gravitational field. Thus the fundamental length postulate is equivalent to gravity field fluctuations.

The existence of quantum fluctuations in the metric [146–148, 160] directly leads to the following conclusion, related to the problem of distance measurements in the Minkowski space: the distance t [7] cannot be measured with precision exceeding [145] the following

$$\delta t = \beta t_{Pl}^{2/3} t^{1/3}, \tag{271}$$

where β is a factor of order of unity. Following [121] we can consider the result (271) as the relation between the UV and IR scales in frames of effective quantum field theory, which correctly describes the entropy features of black holes. Indeed, presenting the relation (252) in terms of length and replacing $\Lambda \to \delta t$, we now recover (271) in holographic context.

The relation (271) together with quantum mechanical energy-time uncertainty relation enables us to estimate the quantum fluctuations energy density in the Minkowski space-time. According to (247) we can consider the region of volume t^3 as composed of cells $\delta t^3 \sim t_{Pl}^2 t$. Therefore such a cell represents the minimally detectable unit of space-time for the scale t. If the age of the chosen region equals t, than it follows from the energy-time uncertainty relations that

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[7] recall that we use the system of units where the light speed equals $c = \hbar = 1$, so that $L_{Pl} = t_{Pl} = M_{Pl}^{-1}$

its existence cannot be realized with energy less than $\sim t^{-1}$. Thus we come to conclusion: if lifetime (age) of some spatial region of linear size t equals t, then there exists a minimal cell δt^3, with energy that cannot be less than

$$E_{\delta t^3} \sim t^{-1}. \tag{272}$$

From (271) and (272) it immediately follows that due to the energy-time uncertainty principle the energy density of quantum fluctuating metric in the Minkowski space equals [146, 148, 160]

$$\rho_q \sim \frac{E_{\delta t^3}}{\delta t^3} \sim \frac{1}{t_{Pl}^2 t^2}. \tag{273}$$

A fact of principal importance is that the dynamical behavior of the metric fluctuations density (273) coincides with the above defined holographic dark energy (252), (255), though the expression were derived based on completely different physical principles. The holographic dark energy density was obtained on the basis of entropy restrictions — the holographic principle, while the metric fluctuations energy density of the Minkowski space is caused only by their quantum nature, namely with the uncertainty principle.

The relation (273) allows to introduce an alternative model for holographic dark energy [160], which uses the age of Universe T for IR-cutoff scale. In such a model

$$\rho_q = \frac{3n^2 M_{Pl}^2}{T^2}, \tag{274}$$

where n is a free parameter of model, and the number coefficient 3 is introduced for convenience. So defined energy density (274) with $T \sim H_0^{-1}$, where H_0 is the current value of the Hubble parameter, leads to the observed value of the dark energy density with the coefficient n value of order of unity. Thus in SCM, where $H_0 \simeq 72 km\,\sec^{-1} Mpc^{-1}$, $\Omega_{DE} \simeq 0.73$, $T \simeq 13.7\,Gyr$, one finds that $n \simeq 1.15$.

Now let us address the crucial question, whether the holographic energy density in form (274) result in accelerated expansion of Universe. For simplicity consider the Universe free of other energy components. In such case the first Friedmann equation reads

$$H^2 = \frac{1}{3M_{Pl}^2}\rho_q. \tag{275}$$

The Universe age T in (274) is linked with scale factor by the relation

$$T = \int_0^a \frac{da'}{Ha'}. \tag{276}$$

Solution of the equation (275) with energy density (274) reads

$$a = \left[n\left(H_0 t + \alpha\right)\right]^n. \tag{277}$$

The integration constant can be determined from the conditions $a_0 = 1$. Evaluating the second order derivative from the scale factor, it can be shown that the accelerated expansion of Universe takes place under the condition $n > 1$. We note that the above obtained value $n \simeq 1.15$, corresponding to the observed value of the dark energy density, satisfies the above mentioned condition, which can be obtained from the conservation equation for dark energy, easily transformed to the following form

$$w_q = -1 - \frac{\dot{\rho}}{3H\rho}. \tag{278}$$

Using (274), (274) and (275) we present the expression (278) in the form

$$w_q = -1 + \frac{2}{3n}. \tag{279}$$

As was repeatedly mentioned above, the accelerated expansion of Universe requires $w < -1/3$, which is equivalent to the above obtained condition $n > 1$. Like in the previous model of the holographic dark energy, we transit to more general case: the Universe where dark energy coexists with matter with density ρ_m. Such Universe is described by the Friedmann equation

$$H^2 = \frac{1}{3M_{Pl}^2}\left(\rho_q + \rho_m\right). \tag{280}$$

Transforming to relative densities $\Omega_m = \frac{\rho_m}{3H^2M_{pl}^2}$ and $\Omega_q = \frac{\rho_q}{3H^2M_{Pl}^2} = \frac{n^2}{T^2H^2}$, we present the Friedmann equation (280) in the form

$$\frac{d\Omega_q}{d\ln a} = (3 - \frac{2}{n}\sqrt{\Omega_q})(1-\Omega_q)\Omega_q. \tag{281}$$

The equation (281) can be solved exactly to give

$$\begin{aligned}\frac{1}{n}\ln a + c_0 \;&=\; -\frac{1}{3n-2}\ln(1-\sqrt{\Omega_q}) - \frac{1}{3n+2}\ln(1+\sqrt{\Omega_q})\\ &+\frac{1}{3n}\ln\Omega_q + \frac{8}{3n(9n^2-4)}\ln(\frac{3n}{2}-\sqrt{\Omega_q}).\end{aligned} \tag{282}$$

The integration constant can be determined from the condition $\Omega_q \simeq 0.73$ at $a=1$. Let us analyze the dynamics of relative dark energy density in two limiting cases: the matter-dominated and dark energy-dominated ones. In the former case it follows from (281) that

$$\Omega_q \approx c_1 a^3. \tag{283}$$

Fast growth of relative contribution of the dark energy, irrespective of n value, results in the dark energy dominated era, when

$$\Omega_q \approx 1 - c_2 a^{-(3n-2)/n}. \tag{284}$$

The state equation for the dark energy can be obtained from the relation (278).

$$w_q = -1 + \frac{2}{3n}\sqrt{\Omega_q}. \tag{285}$$

In the early Universe during matter-dominated era one gets $\Omega_q \to 0$ and $w_q \to -1$, i.e. during that period the holographic dark energy in the considered model behaves like cosmological constant. In later epoch of dark energy domination, when $\Omega_q \to 1$, the state equation (285) naturally transforms to the above obtained relation (279). Remark that fate of the Universe filled by matter and holographic dark energy with density (274) is constant accelerated expansion with power law time dependence (277) of the scale factor. Thus the holographic model for dark energy with IR-cutoff scale set to the Universe age, allows the following:

1. to obtain the observed value of the dark energy density;
2. provide the accelerated expansion regime on later stages of the Universe evolution;
3. resolve contradictions with the causality principle.

However the reader should not hurry with the ultimate conclusions. The first successes of holographic principle application from one hand awoke hopes to create on that basis an adequate description of the Universe dynamics, free of a number of problems which the traditional approach suffers from. From the other hand, it was those successes which provoke we would say unreasonable optimism. We believe that papers entitled somewhat like "Solution of dark energy problem" [200] represent manifestation of specific "holographic extremism". We would like to remind the saying of W.Churchill: "Success means motion from failure to failure, not loosing the enthusiasm". The holographic dynamics is one of the youngest directions of theoretical physics. On that path the physicists experienced yet too few failures in order to pretend for complete success.

X. TRANSIENT ACCELERATION

Unlike the fundamental theories, the physical models only reflect our current understanding of a process or a phenomenon, to describe which they were created. The model efficiency significantly depends on its flexibility, i.e. its ability to be modernized using the new-coming information. That is why evolution of any actively living model includes multiple generalizations, directed both to resolve the conceptual problems and to describe ever growing set of observational data. In the case of SCM those generalizations can be divided on two types. The first includes replacements of cosmological constant by more complicated dynamical forms of dark energy, taking into account the possibility of interaction of the latter with dark matter. Generalizations of the second type are of more radical nature and pretend one consequent change of cosmological paradigm. The ultimate (explicit or hidden) aim of them is to

reject the dark components due to modifications of Einstein equations, and consequently the Newtonian laws two. The generalizations of both types can be well viewed on example of the phenomenon called the “transient acceleration”.

As we have seen above in frames of SCM, the dependence of the deceleration parameter q on the redshift z has the characteristic monotonous trend to the limiting value $q(z) = -1$ for $z \to -1$. It physically means that after the beginning of dark energy-dominated era (at $z \sim 1$), the Universe in SCM is condemned to eternal acceleration.

Below we consider some cosmological models with dynamical forms of dark energy, which lead to transient acceleration, and we also discuss the information on current Universe expansion rate extracted from observational data.

A. Theoretical background

J. Barrow [38] was one of the first to consider principal possibility of the transient acceleration. He showed that many well established scenarios, consistent with the current accelerated expansion of Universe, do not exclude the possibility of recurrence to non-relativistic matter-dominated era, and thus to the decelerated expansion regime. Therefore the transition to the accelerated expansion does not yet imply the eternal accelerated expansion.

In order to show that we, following the Barrow work [38], consider a homogeneous and isotropic flat Universe, filled by non-relativistic matter and scalar field with the potential energy $V(\varphi)$ and state equation $p = w\rho$. We take the scalar field potential in the form

$$V(\varphi) = V_p(\varphi)e^{-\lambda\varphi}. \tag{286}$$

In some versions of low-energy limits of string theory the potential $V_p(\varphi)$ represents a polynomial. The exponential potential with a shallow minimum was first suggested by Albrecht and Skordis [37]. This minimum on the exponential potential background was created due to the polynomial factor $V_p(\varphi)$ of the simplest form

$$V_p(\varphi) = (\varphi - \varphi_0)^2 + A. \tag{287}$$

In the considered case the potential takes the following form

$$V(\varphi) = e^{-\lambda\varphi}\left(A + (\varphi - \varphi_0)^2\right). \tag{288}$$

In order to relate the above considered potential to that of string theory the constant parameters A and φ_0 should be of order of unity in Planck units. In such quintessence models the accelerated expansion of Universe appears naturally on late stages of evolution without any fitting of initial parameters, thus this model is free of the precise tuning problem.

The accelerated expansion starts when the field rolls down the local potential minimum at $\phi = \phi_0 + (1 \pm \sqrt{1-\lambda^2 A})/\lambda$, which is formed by the quadratic factor in (288), where $1 \geq \lambda^2 A$. While the field stays in the false vacuum state its kinetic energy is negligibly small ($\phi \approx const$), and consequent domination ρ_ϕ is caused by almost constant value of the potential energy, which runs the era of the accelerated expansion of Universe which never ends.

It was discovered in the paper [38] that it is neither solely possible nor most probable scenario.

The transient vacuum domination appears in the two following cases. When $A\lambda^2 < 1$, the field φ reaches the local minimum with kinetic energy sufficient to overcome the potential barrier and continues roll down the exponential part of the potential to the region where $\varphi \gg \varphi_0$. The kinetic energy is defined by the scaling regime and thus by the parameters of potential rather than by the initial conditions. The transition acceleration appears also in the case when the condition $A\lambda^2 > 1$ holds. Since A grows proportional to λ^{-2}, the potential looses its local minimum and flattens near the inflection point. It is sufficient to cause the temporal acceleration of the Universe expansion, however the field never stops rolling down the potential and once the Universe will be again dominated by matter with the dependence $a(t) \propto t^{2/3}$.

Therefore the Universe quits the regime of eternal accelerated expansion and returns to the decelerated expansion. Besides the the well motivated family Albrecht-Skordis potentials the possibility of transient acceleration regime appears to be more probable than the eternal accelerated expansion.

B. Different models with transient acceleration

In order to show explicitly that the transient acceleration represents a natural feature of different cosmological models, we briefly consider some of them below.

Barrow considered in his work [38] a model of Universe where dark energy is present in form of the scalar field with potential (288). We consider some other examples of cosmological models that provide alteration of the accelerated expansion phase by the decelerated one.

In the paper [66] it was shown that the transition acceleration phase can be realized in the exponentail potential as well, and besides that a d-dimensional cosmological model was also considered in the work. The action of the latter reads

$$S = \int d^d x \sqrt{-g} \Big(\frac{R}{2\kappa^2} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_0 \exp(-\lambda\phi) \Big), \tag{289}$$

where $\kappa_N^2 = 8\pi G_N = 1/2$ and $V_0 > 0$, $\lambda > 0$ (the case $\lambda < 0$ is connected to the case $\lambda > 0$ by the replacement $\phi \to -\phi$). Following the paper [66] we consider below the FRW-metric for flat Universe ($k = 0$)

$$ds^2 = dt^2 - a^2(t) dx^i dx^i \ , \qquad i = 1, ..., d-1. \tag{290}$$

In that case the action takes the form

$$S = \int d^d x \Big((d-1)(d-2) a^{d-3} \dot{a}^2 + a^{d-1} \big(\frac{1}{2} \dot{\phi}^2 - V(\phi) \big) \Big), \tag{291}$$

and in variables (u, v)

$$\phi = \frac{1}{\kappa} \sqrt{\frac{d-2}{d-1}} (v - u) \ , \quad a^{d-1} = e^{v+u}, \tag{292}$$

one obtains

$$\begin{aligned} S &= \int d^{d-1} x dt \ e^{u+v} \Big(\frac{2(d-2)}{\kappa^2 (d-1)} \dot{u} \dot{v} - V_0 e^{-2\alpha(v-u)} \Big), \\ \alpha &\equiv \frac{1}{2\kappa} \sqrt{\frac{d-2}{d-1}} \lambda. \end{aligned} \tag{293}$$

Let us transform to new time variable τ

$$\frac{d\tau}{dt} = \kappa \sqrt{\frac{(d-1) V_0}{2(d-2)}} \ e^{\alpha(u-v)}. \tag{294}$$

and therefore

$$S = \frac{1}{\kappa} \sqrt{\frac{2(d-2) V_0}{d-1}} \int d^{d-1} x d\tau \ e^{u+v} e^{\alpha(u-v)} \big(u' v' - 1 \big).$$

Using (293), (294) and (292) it can be shown [66], that the general solution for the case $\alpha < 1$ takes the form:

$$\begin{aligned} d\mathfrak{s}^2 &= \frac{2(d-2)}{\kappa^2 (d-1) V_0} \ e^{\frac{4\alpha^2 \tau}{w}} \frac{(1 + m e^{-2w\tau})^{\frac{2\alpha}{(1-\alpha)}}}{(1 - m e^{-2w\tau})^{\frac{2\alpha}{(1+\alpha)}}} d\tau^2 - e^{\frac{4\tau}{sw}} (1 + m e^{-2w\tau})^{\frac{2}{s(1-\alpha)}} (1 - m e^{-2w\tau})^{\frac{2}{s(1+\alpha)}} dx^i dx^i; \\ \phi &= \frac{1}{\kappa} \sqrt{\frac{d-2}{d-1}} \Big(\frac{2\alpha\tau}{w} - \frac{1}{1+\alpha} \log(1 - m e^{-2w\tau}) + \frac{1}{1-\alpha} \log(1 + m e^{-2w\tau}) \Big), \end{aligned} \tag{295}$$

where $s = d - 1$, and m is the integration constant.

The solution (295) has the asymptotes:

$$\begin{aligned} & a \sim t^{\frac{1}{d-1}} \ , \quad \phi = -\frac{1}{\kappa} \sqrt{\frac{d-2}{d-1}} \log t \ , \quad \text{for} \quad t \cong 0 \ , \\ & a \sim t^{\frac{4\kappa^2}{(d-2)\lambda^2}} \ , \quad \phi = \frac{2}{\lambda} \log t \ , \qquad \text{for} \quad t \gg 1 \ . \end{aligned} \tag{296}$$

On early stages of evolution the state equation is extremely rigid $p = \rho$, and on later stages it is easy to show that

$$p = \omega\rho, \qquad \omega = \frac{d-2}{2\kappa^2(d-1)} \lambda^2 - 1 \ .$$

In accordance with the with the equation (296), the accelerated expansion will continue forever under condition $\lambda < \frac{2\kappa}{\sqrt{d-2}}$. Let us show that under condition $\frac{2\kappa}{\sqrt{d-2}} < \lambda < 2\kappa\sqrt{\frac{d-1}{d-2}}$ the solution (295) with $m > 0$ has a transient acceleration stage. First let us find $\frac{da}{dt} = \frac{da}{d\tau}\frac{d\tau}{dt}$. Using (295) it is easy to show that $\dot{a}$ is proportional to positively defined quantities m and τ. Setting $|m| = 1$, which can be easily done by shift along the τ axis, one finds $\ddot{a}$ in the following form:

$$\ddot{a} = -(\text{positive})\Big(\big((d-1)\alpha^2 - 1\big)Z^2 - 2(d-2)\,\mathrm{sign}(m)\alpha Z + d - 1 - \alpha^2 \Big), \tag{297}$$

where $Z \equiv \cosh(2w\tau)$. If $(d-1)\alpha^2 < 1$, which corresponds to the case $\lambda < \frac{2\kappa}{\sqrt{d-2}}$, then we obtain the eternal accelerated expansion for arbitrary values of m, because only the first term in (297)dominates on the later stages of evolution. Remark that such solution represents an attractor. If $(d-1)\alpha^2 > 1$ holds for later times then the solution always corresponds to to decelerated expansion of Universe. For $(d-1)\alpha^2 > 1$ and $m < 0$ the right hand side of the equation (297) is negatively defined, which corresponds to decelerated expansion of Universe during whole evolution time. At last, in the case $(d-1)\alpha^2 > 1$ and $m > 0$, the solution always provides the transient acceleration phase, which is mentioned in[67]. Indeed, the equation (297) has two roots

$$Z_\pm = \frac{(d-2)\alpha \pm \sqrt{d-1}(1-\alpha^2)}{(d-1)\alpha^2 - 1}, \tag{298}$$

defined in the interval $\tau_-(\alpha) < \tau < \tau_+(\alpha)$, corresponding to the accelerated expansion.

The root $\tau_\pm$ is real (and positive), because on the interval $\alpha \in (1/\sqrt{d-1}, 1)$ we have $Z_\pm > 1$. In the limit $\alpha \to 1$ the two roots coincide and duration of the accelerated expansion phase tend to zero. In the opposite limit $\alpha \to 1/\sqrt{d-1}$ one obtains $Z_- = d/(2\sqrt{d-1})$ and $Z_+ \to \infty$, which corresponds to infinite period of accelerated expansion. For higher dimensions of space one obtains $Z_\pm = 1/\alpha \pm (1-\alpha^2)/(\alpha^2\sqrt{d}) + O(1/d)$, and it means that the transient acceleration duration is shorter in the Universe with larger number of spatial dimensions.

2. *Transient acceleration in models with multiple scalar fields*

The existence of an event horizon in the case of a de Sitter phase is an obstacle to the implementation of string theory because the S-matrix formulation is no longer possible, hence eternal acceleration, leading to a de Sitter space in the asymptotic future is problematic [70]. The transient acceleration models allow to avoid that contradiction, because they produce the accelerated expansion today and in recent past, without any contradiction in future. In the paper [68] the dynamics of homogeneous and isotropic Universe was considered, where the dark energy is realized by two (generally speaking, turn by turn) scalar fields Φ and ψ. Let us write down the equations of motion for that system

$$\begin{aligned} \dot{\rho}_b &= -3H\gamma_b\rho_b \\ \ddot{\varphi} &= -3H\dot{\varphi} - \partial_\varphi V \\ \ddot{\psi} &= -3H\dot{\psi} - \partial_\psi V, \end{aligned} \tag{299}$$

and also the first Friedmann equation:

$$H^2 = \frac{8\pi G}{3}(\rho_b + \rho_Q) - \frac{k}{a^2}\ .$$

Here the dot denoted the derivative with respect to time t, the subscript b marks the background components such as dark matter (m) or radiation (r), and Q stands for dark energy in form of the two scalar fields. It is convenient also to define

$$p_{b,Q} = (\gamma_{b,Q} - 1)\rho_{b,Q}\ ,$$

then $\gamma_m = 1$ and $\gamma_r = \frac{4}{3}$, where, for any component i, we have introduced for convenience the quantity $\gamma_i \equiv 1 + w_i$. The energy density and pressure for the quintessence fields in the potential $V(\varphi,\psi)$ have the form:

$$\rho_Q = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}\dot{\psi}^2 + V(\varphi,\psi) \quad p_Q = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}\dot{\psi}^2 - V(\varphi,\psi).$$

As always, the possibility of the transient acceleration crucially depends on the potential form. Below we consider several possibilities to obtain the transient acceleration. The paper [68] considers a number of examples, including the case when the two fields interact through the coupling potential $V(\varphi,\psi)$, as well as the case when one of the fields is free. Here we consider only the models with potentials depending on both fields. Direct generalization of the potential from the paper (288) for the case of minimal coupling between the two scalar fields enables us to obtain the transient acceleration.

Consider a potential of the form

$$V(\varphi,\psi) = M^4 e^{-\lambda\varphi}(P_0 + f(\psi)(\varphi-\varphi_c)^2 + g(\psi)). \tag{300}$$

The above introduced additional scalar field ψ will control absence or presence o the potential minimum for φ. The main idea is that, from one hand, a potential minimum initially exists for the scalar φ, which provides accelerated expansion of Universe, and from the other hand evolution of the field ψ results in the extinction of the minimum and the Universe returns to the decelerated expansion regime. The model of [37] is recovered with $f \equiv 1$ and $g \equiv 0$. For the potential (300) the position of the minimum is determined by the expression

$$\varphi_\pm = \varphi_c + \frac{1}{\lambda}\left(1 \pm \sqrt{1-\lambda^2\frac{P_0+g(\psi)}{f(\psi)}}\right). \tag{301}$$

The function g $(g>0)$ is responsible for the scalar field mass and it usually takes the form $g \propto \psi^2$, but as it does not affect the system dynamics, we set for simplicity $g \equiv 0$.

The minimum (301) disappears under the condition

$$f(\psi) < \lambda^2 P_0 \equiv f(\psi_c). \tag{302}$$

Note that the potential (300) can be presented in the form

$$V(\varphi,\psi) = M^4/\lambda^2 e^{-\lambda\varphi}(f(\psi_c) + f(\psi)(\lambda\varphi-\lambda\varphi_c)^2).$$

We assume that f is positive, continuous and monotonous function for $\psi > 0$ and/or $\psi < 0$. From the condition (340) it is easy to see that if f decreases (increases), then for $\psi < \psi_i$ (ψ_i- is the initial value of the scalar field), $(\psi < \psi_c)$ the accelerated expansion takes place, because the minimum (301) exists. One is free to use different functions f to obtain the transient acceleration, for instance we present the case $p,\ \alpha \geq 0,\ f = 1+\alpha\psi^p,\ f = \tanh(\alpha\psi^p)$, or for arbitrary p and α, $f = \exp(\alpha\psi^p),\ f = \cosh(\alpha\psi^p)$. During the evolution of Universe the scalar field φ rolls down the potential and dominates in the exponential part so that $M_{Pl}^2 m_\varphi^2 \sim M_{Pl}^2 m_\psi^2 \sim V \sim \dot\varphi^2 \sim M_{Pl}^2 H^2$ with $\Omega_Q = 4/\lambda^2$, $w_Q = 1/3$ during the radiation dominated era, and with $\Omega_Q = 3/\lambda^2$, $w_Q = 0$ for the matter-dominated one, while φ approach the minimum. When the field φ is in the minimum (301) and oscillates near its minimum value, the universe expands with acceleration with $V \gg \dot\varphi^2$ and therefore $w_Q \simeq -1$ until ψ obeys $\psi \leq \psi_c$, $(\psi \geq \psi_c)$, if the function f grows (decays). In that moment the minimum (301) disappears and the field φ starts free roll acquiring large values (speeding up) and the the Universe expansion becomes decelerated one $q > 0$.

When ψ is initially larger (smaller) than ψ_c, provided f is increasing (decreasing) ψ passes through ψ_c because $\partial_\psi V = M^4 e^{-\lambda\phi}(\phi-\phi_c)^2\frac{df}{d\psi}$ is positive (negative), acceleration occurs which is always transient; if on the contrary $\psi_i \leq \psi_c$ $(\psi_i \geq \psi_c)$ quintessence domination is not possible. The critical value ψ_c controls the presence or not of the minimum for ϕ.

For a particular example let us consider the simplest case

$$f(\psi) = \psi^2\ , \tag{303}$$

then the minimum for φ disappears under condition $-\psi_c \leq \psi \leq \psi_c$, where $\psi_c \equiv \lambda\sqrt{P_0}$. In analogy with the model (288), one can obtain $\lambda \gtrsim 9$, which agrees with the limitations imposed by the cosmological observations, provided the initial value φ_i is fixed and the critical value φ_c must be finely tuned in order to make the quintessence dominate in the present time.

The diagram 9 shows the state equation parameter $w_{Q,0} \simeq -0.491$ and $w_{eff,0} = -0.874$. When the field φ gets into the minimum, then $w_Q \simeq -1$. The deceleration parameter q is also shown on the plot, and it points out on the fact that in the present time the Universe already passed the transient acceleration stage and currently it expands with deceleration $q_0 \simeq 0.013$. The accelerated expansion $(q<0)$ starts at $z \simeq 0.658$ and stops at $z \simeq 0.0035$, when the Universe age equals $t_{end}/t_0 \simeq 0.996$, while in the present time one has $H_0 t_0 \simeq 0.912$.

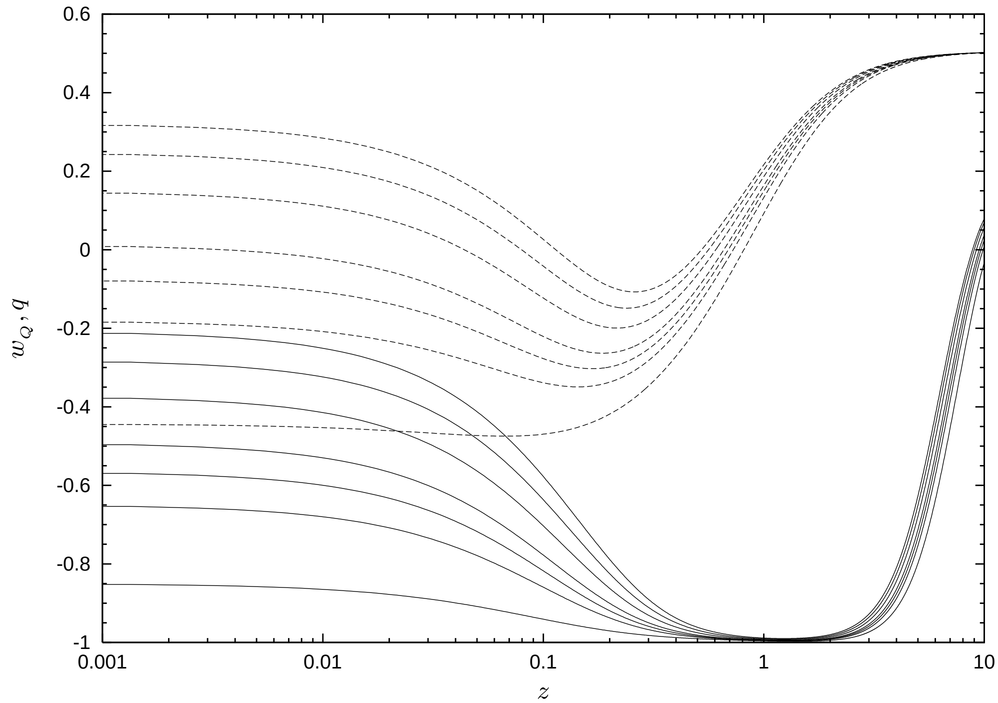

FIG. 9: The diagram plots the dependence of the state equation parameter w_Q (solid line) and deceleration parameter q (dashed line) as functions of the redshift z for the potential (300) with the fixed parameter values $\lambda = 10$, $\phi_c = 23.8$ and P_0 taking the following values values (upward from below) $0.160, 0.162, 0.163, 0.164, 0.166, 0.168, 0.170$ ($\phi_i = 0$ and $\psi_i = 5$). In the models with $p_0 = 0.164, 0.166, 0.168, 0.170$ the accelerated expansion is replaced by the decelerated one in the modern epoch.

3. *Decaying dark energy as a scalar field*

As was already mentioned above, the cosmological models where the dark energy decays have many attractive features, one of which being the presence of transient acceleration — alteration of the accelerated Universe expansion period by the decelerated one, which takes place when the dark energy density becomes sufficiently low. Dynamics of such Universe clearly differs from its evolution predicted by SCM. The considered model with the decaying dark energy represents a pre-image of the commonly accepted inflation model, where the field, generated the inflationary expansion of Universe, experiences the decay.

Consider a scalar field model with the potential that takes both positive and negative values [107]:

$$V(\varphi) = V_0 \cos\frac{\varphi}{f}, \quad f = \frac{\sqrt{V_0}}{m}. \tag{304}$$

The assumption that the scalar field potential can take negative values is very curious, but one should remember that except the motivation to explain the observed transient acceleration this model lacks any other observational support. Nevertheless such effective potentials often appear in the super-gravity and M-theory. It is be shown below how to obtain the transient acceleration in frames of that model. Remark also that evolution of Universe in cosmological models with negative potentials sharply differs from that of SCM. So, for example, in the case when the Friedmann equation is dominated by potential energy (304), taking negative values

$$\left(\frac{\dot a}{a}\right)^2 = \frac{1}{3}\left(\rho + \frac{1}{2}\dot\varphi^2 + V(\varphi)\right), \tag{305}$$

even the spatially flat Universe can collapse, which is principally impossible in SCM, but it can happen only on later stages of Universe evolution, far before the transient acceleration ends. Up to the moment when the Universe starts collapsing, many stages of transient acceleration manage to occur (see Fig.10), which become more often when approaching the collapse moment.

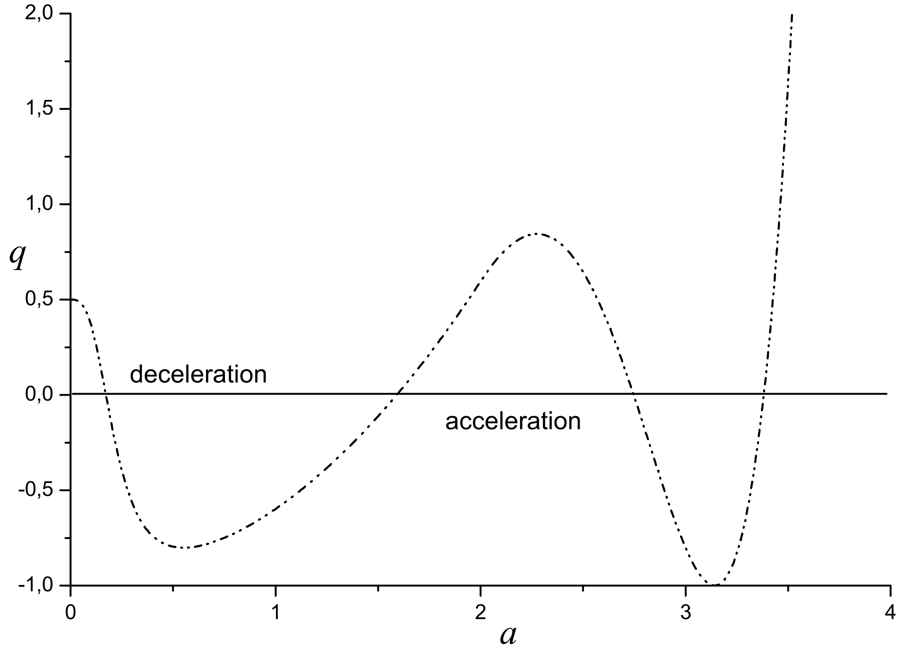

FIG. 10: Dependence of the deceleration parameter on the scale factor for the Universe filled by matter and dark energy in form of scalar field in the potential $V(\varphi) = V_0 \cos\left(\varphi m/\sqrt{V_0}\right)$, with $m = 0.74$; $V_0 = 150, \varphi(0) = 0.23, \varphi'(0) = 0$. The present time corresponds to $a = 1$. The potential parameters are chosen so that the deceleration parameter value appears to be the same as in SCM, $q(1) \approx -0.6$.

4. Transient Acceleration In the Universe with the Interacting Components

Consider the spatially flat Universe composed from three components [101]: dark energy, dark matter and baryons. The first friedmann equation then takes the form:

$$3H^2 = \rho_{DE} + \rho_m + \rho_b, \tag{306}$$

where as usual ρ_{DE} is the dark energy density, ρ_m – dark matter energy density, ρ_b – the density of baryons, $H = \frac{\dot{a}}{a}$ – Hubble parameter. The state equation for the dark energy takes the form $p_{DE} = w\rho_{DE}$.

The conservation equations in that case take the form:

$$\begin{aligned} \dot{\rho}_{DE} + 3H(1+w)\rho_{DE} &= -Q; \\ \dot{\rho}_m + 3H\rho_m &= Q, \end{aligned} \tag{307}$$

where Q characterizes the interaction. The conservation equation for the baryons reads:

$$\dot{\rho}_b + 3H\rho_b = 0 \;\;\Rightarrow\;\; \rho_b = \rho_{b0}\left(\frac{a_0}{a}\right)^3. \tag{308}$$

Total density equals $\rho = \rho_m + \rho_b + \rho_{DE}$. Without lack of generality, we assume that the dark matter energy density satisfies

$$\rho_m = \tilde{\rho}_{m0}\left(\frac{a_0}{a}\right)^3 f(a), \tag{309}$$

where $\tilde{\rho}_{m0}$ and a_0 are constants and $f(a)$ is arbitrary time dependent function. From (307) and (309) one obtains

$$Q = \rho_m \frac{\dot{f}}{f} = \tilde{\rho}_{m0}\left(\frac{a_0}{a}\right)^3 \dot{f}. \tag{310}$$

Let $f(a)$ takes the form

$$f(a) = 1 + g(a). \tag{311}$$

Interaction free case corresponds to $f(a)=1$, thus the function $g(a)$ responds for the interaction. Taking into account that

$$\dot{f}=\dot{g}=\frac{dg}{da}\dot{a}, \tag{312}$$

one obtains

$$Q=\tilde{\rho}_{m0}\frac{dg}{da}\dot{a}\left(\frac{a_0}{a}\right)^3. \tag{313}$$

For ρ_m it means that

$$\rho_m=\tilde{\rho}_{m0}\left(1+g\right)\left(\frac{a_0}{a}\right)^3, \tag{314}$$

where $\rho_{m0}=\rho_m(a_0)$ in presence of interaction, and analogously $\tilde{\rho}_{m0}$ in uncoupled case. The two initial values of dark matter density are linked by the following relation

$$\rho_{m0}=\tilde{\rho}_{m0}\left(1+g_0\right), \tag{315}$$

where $g_0\equiv g(a_0)$. As can be seen from (310), in the case when Q is positive the dark energy decays into dark matter $\frac{dg}{da}>0$ and vice versa $\frac{dg}{da}<0$ otherwise. It follows from the equations (307) and (313)

$$\dot{\rho}_{DE}+3H\left(1+w\right)\rho_{DE}=-\tilde{\rho}_{m0}\frac{dg}{da}\dot{a}\left(\frac{a_0}{a}\right)^3. \tag{316}$$

Assuming that $w=const$ the solution of equation (316) takes the form

$$\begin{gathered}\rho_{DE}=\left(\rho_{DE0}+\tilde{\rho}_{m0}g_0\right)\left(\frac{a_0}{a}\right)^{3(1+w)}-\\ -\tilde{\rho}_{m0}\left(\frac{a_0}{a}\right)^3 g+3w\tilde{\rho}_{m0}a_0^3a^{-3(1+w)}\int_{a_0}^{a}da\,ga^{3w-1}.\end{gathered} \tag{317}$$

Rewrite the second Friedmann equation in the terms of $g(a)$

$$\begin{gathered}\frac{\ddot{a}}{a}=-\frac{1}{6}\left\{\tilde{\rho}_{m0}\left(1+g\right)\left(\frac{a_0}{a}\right)^3+\rho_{b0}\left(\frac{a_0}{a}\right)^3+\left(1+3w\right)\times\right.\\ \left.\times\left[\left(\rho_{DE0}+\tilde{\rho}_{m0}g_0\right)\left(\frac{a_0}{a}\right)^{3(1+w)}-\tilde{\rho}_{m0}\left(\frac{a_0}{a}\right)^3 g+3w\tilde{\rho}_{m0}a_0^3a^{-3(1+w)}\int_{a_0}^{a}da\,ga^{3w-1}\right]\right\}.\end{gathered} \tag{318}$$

In order to solve the equation (318) one have to explicitly define the function $g(a)$. For the lack of knowledge about nature of the dark energy, as well as the dark matter, the function $g(a)$ cannot be derived from the first principles. Therefore for the model under consideration we introduce the interaction such that the resulting dynamics in the model corresponds to the observed data.

Consider interaction with the function $g(a)$ in the form $g\left(a\right)=a^n\exp\left(-a^2/\sigma^2\right)$, where n is natural number, and σ is positive real number. The transient acceleration implies that the dark energy density starts to diminish, i.e. experiences decay $\frac{dg}{da}>0$. This condition requires that n and σ satisfy the inequality $n\sigma^2>2$. The dependencies of relative densities on the scale factor for $n=7$ and $\sigma=1.5$ are shown on Fig. 11. The considered model which allows to produce the transient acceleration with certain choice of the interaction parameters, however for large values of scale factor as well for small ones it is indistinguishable from SCM.

5. *Decaying cosmological constant and transient acceleration phase*

As a simple example of the transient acceleration we consider a model [101] with decaying cosmological constant:

$$\dot{\rho}_{dm}+3\frac{\dot{a}}{a}\rho_{dm}=-\dot{\rho}_\Lambda, \tag{319}$$

where ρ_{dm} and ρ_Λ are energy densities of dark matter and cosmological constant Λ respectively. On early stages of Universe expansion, when ρ_Λ is sufficiently small, such decay has almost no influence on the cosmic evolution. On later stages, the more the dark energy contribution grows, the more its decay affects the standard dependence for the

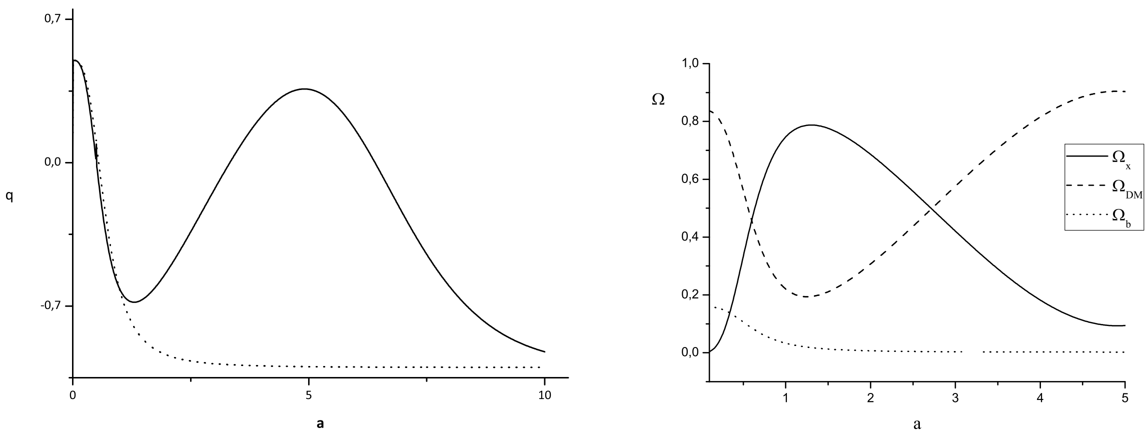

FIG. 11: The dependencies of relative densities on the scale factor for $n = 7$ and $\sigma = 1.5$ (on the left). On the right: the dependencies of deceleration parameter on the scale factor in the model with interacting dark energy and dark matter $q(a)$(solid line) with $n = 7$ and $\sigma = 1.5$ in comparison to SCM (dashed line).

dark matter energy density $\rho_{dm} \propto a^{-3}$ on the scale factor a. Let us assume that such deviation can be described by a scale factor function $\epsilon(a)$, then

$$\rho_{dm} = \rho_{dm,0} a^{-3+\epsilon(a)} \,, \tag{320}$$

where in the present epoch $a_0 = 1$. Other material fields (radiation and baryons) evolve independently and conserve. Therefore the dark energy density takes the form:

$$\rho_\Lambda = \rho_{dm,0} \int_a^1 \frac{\epsilon(\tilde{a}) + \tilde{a}\epsilon' \ln(\tilde{a})}{\tilde{a}^{4-\epsilon(a)}} d\tilde{a} + \mathrm{X}, \tag{321}$$

where prime denotes the derivative with respect to the scale factor, and X − is the integration constant. Neglecting the contribution of radiation, the first Friedmann equation takes the form

$$H = H_0 \left[\Omega_{b,0} a^{-3} + \Omega_{dm,0}\varphi(a) + \Omega_{\mathrm{X},0}\right]^{1/2}. \tag{322}$$

The function $\varphi(a)$ reads

$$\varphi(a) = a^{-3+\epsilon(a)} + \int_a^1 \frac{\epsilon(\tilde{a}) + \tilde{a}\epsilon' \ln(\tilde{a})}{\tilde{a}^{4-\epsilon(a)}} d\tilde{a}, \tag{323}$$

where $\Omega_{\mathrm{X},0}$ denotes relative contribution of the constant X into the total relative density. In order to proceed further, some assumptions on the explicit form of the function $\epsilon(a)$ should be done. In the present review we follow the original paper [101] on Λ-dark matter interaction and consider the simplest case

$$\begin{aligned} \epsilon(a) &= \epsilon_0 a^\xi; \\ &= \epsilon_0 (1+z)^{-\xi}, \end{aligned} \tag{324}$$

where ϵ_0 and ξ can take both positive and negative values. From the above cited expression (321) it follows that

$$\rho_\Lambda = \rho_{m0}\epsilon_0 \int_a^1 \frac{[1 + \ln(\tilde{a}^\xi)]}{\tilde{a}^{4-\xi-\epsilon_0 \tilde{a}^\xi}} d\tilde{a} + \mathrm{X}. \tag{325}$$

Remark that the case $\epsilon_0 = 0$ corresponds to SCM, i.e. $\mathrm{X} \equiv \rho_{\Lambda 0}$.

Using the above cited formulae it is easy to obtain the dependence of the relative densities $\Omega_b(a)$, $\Omega_{dm}(a)$ and $\Omega_\Lambda(a)$:

$$\Omega_b(a) = \frac{a^{-3}}{\mathrm{A} + a^{-3} + \mathrm{B}^{-1}\varphi(a)}; \tag{326a}$$

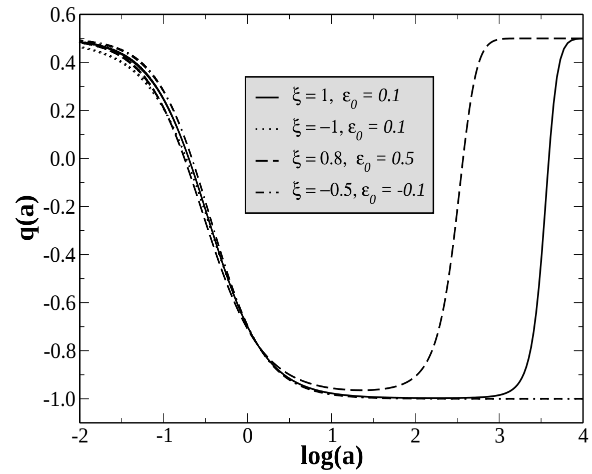

FIG. 12: The deceleration parameter as function of $\log(a)$ for some selected values of ϵ_0 and ξ.

$$\Omega_{dm}(a) = \frac{a^{-3+\epsilon(a)}}{\mathrm{D} + \mathrm{B}a^{-3} + \varphi(a)}; \tag{326b}$$

$$\Omega_{\Lambda}(a) = \frac{\mathrm{D} + \varphi(a) - a^{-3+\epsilon(a)}}{\mathrm{D} + \mathrm{B}a^{-3} + \varphi(a)}; \tag{326c}$$

where $\mathrm{A} = \Omega_{\mathrm{X},0}/\Omega_{b,0}$, $\mathrm{B} = \Omega_{b,0}/\Omega_{dm,0}$ and $\mathrm{D} = \Omega_{\mathrm{X},0}/\Omega_{dm,0}$.

In such simple model, appropriate choice of the parameters ϵ_0 and ξ enables to obtain practically any kind of Universe dynamics. In the context of the present review it is especially interesting to consider the case when $\epsilon_0 > 0$ and ξ takes large positive values ($\xi \gtrsim 0.8$). The figure 12 shows the dependence of the scale factor for the case $\xi = 1.0$ and $\epsilon_0 = 0.1$. Note that with such parameters in the present time $a \sim 1$ the Universe expands with acceleration, but unlike the case of SCM, the dark energy domination will not last forever and at $a \gg 1$ the Universe enters into new era of non-relativistic matter domination with $a \to \infty$. Such type of dynamical behavior is impossible for most models with $\Lambda(t)$ or those with interacting quintessence, widely discussed in literature, but it is an attribute of the so-called thawing [40] and hybrid [41] potentials, which follow from string or M-theory [42] (see also [43])[8].

In order to give clearer idea of transient acceleration phenomenon we find the explicit form of the deceleration parameter $q = -a\ddot{a}/\dot{a}^2$, in the considered model

$$q(a) = \frac{3}{2}\frac{\Omega_{b,0}a^{-3} + \Omega_{dm,0}a^{\epsilon(a)-3}}{\Omega_{b,0}a^{-3} + \Omega_{dm,0}\varphi(a) + \Omega_{\mathrm{X},0}} - 1. \tag{327}$$

Its dependence on $\log(a)$ for selected values of ξ and ϵ_0 is shown on Fig.12. Note that large values of the parameter ξ corresponds to the Universe dominated by matter in the past ($q(a) \to 1/2$ for $a \gg 1$), afterwards (at $a_{acc} < 1$) a long-during era of accelerated expansion takes place, which, unlike SCM, does not last forever, ending at some $a_{dec} > 1$.

C. Transient acceleration: holographic limitations

The novel concept of the holographic Universe considered in the previous section turns out to impose strict limitations to Universe dynamics. In order to see that we consider the matter entropy contained within the observed

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[8] Authors of the paper [42] present arguments for the fact that forever expanding Universe, which is common feature of most quintessence models (including the standard ΛCDM-model), contradicts the predictions of string/M-theories, because those models contain cosmological horizon, which make impossible to apply the usual procedure of S-matrix formalism, describing the interaction of the particles.

horizon in the case when the matter is composed of non-relativistic substance. Each particle of the substance carries one unit of information, and therefore $S_m = K_B N$, where $N = (4\pi/3) r_A^3 n$, and n is the particle concentration, for which $n = n_0 a^{-3}$. Thus one obtains for entropy the following

$$S_m = k_B \frac{4\pi}{3} \tilde{r}_A^3 \, n_0 \, a^{-3} \propto a^{3/2} \,. \tag{328}$$

and it follows that $S_m'' \propto \frac{3}{4\sqrt{a}} > 0$. It is interesting to note that as the Universe expands the particle number contained inside the horizon increases too, which provides the entropy growth and therefore $S_m'' + S_A'' > 0$. It is easy to see that the Universe tends to maximum entropy provided its horizon decreases with time. In the simplest case the horizon equals to Hubble radius and the above condition is satisfied if the Universe expands with acceleration. In such case the deceleration parameter is negative ($q < 0$), thus

$$\frac{d}{dt}(R_H) = c(1+q) < c,$$

and Hubble sphere has velocity which is less then light speed by the quantity cq, and thus it is left behind those galaxies. Therefore the particles initially placed inside the Hubble sphere, gradually leave it.

From the latter fact that a conclusion follows that in order to obey the second law of thermodynamics, the universe must be dominated by dark energy with state equation parameter satisfying the condition $-1 \le w \le -2/3$. Another alternative is to modify the gravitation theory in order to obtain the accelerated dynamics without the dark energy.

Likewise, since every isolated system is expected to evolve in a such a way that it tends to thermodynamic equilibrium the curvature of the $\mathcal{A}(a)$ function should never spontaneously change from negative to positive values. In order to specify the set of possible cosmological values, or equivalently to impose certain limitations on the values taken by the state equation parameter w, it is necessary to use the laws which are not immediately connected to cosmology. In the context of holographic cosmology it is reasonable to use the generalized law of the black hole thermodynamics — the generalized square law (GSL). According to the latter the total entropy of the system S must obey the condition $S'' < 0$. As will be shown below, (see [104]) application of this criterion on later stages of Universe evolution helps considerable specify the range of values allowable for the state equation parameter w.

For further convenience we express the horizon area $\mathcal{A}$ and its derivatives through the deceleration parameter $q \equiv -\ddot{a}/(a\,H^2)$, which can be presented in the above mentioned form:

$$q = -\left(1 + \frac{a\,H'}{H}\right) . \tag{329}$$

For the case of spatially flat Universe $\mathcal{A} = 4\pi H^{-2}$, therefore $\mathcal{A}' = 8\pi(1+q)/H^2$ and

$$\mathcal{A}'' = 2\mathcal{A}\left[(1\,+\,q)\,(1\,+\,2q)\,+\,\frac{q'}{a}\right] . \tag{330}$$

It follows that for the case $q \ge -1/2$ the second order derivative of the horizon area, $\mathcal{A}''$, cannot change from positive to negative values while $q' < 0$. It excludes the dependence for q shown on Fig.13. Such dependence of the deceleration parameter appears in the cosmological models where the current accelerated expansion is transient, and after a short dark matter dominated period the dark matter starts dominating again, which leads to the decelerated Universe expansion era. [105, 106].

D. Transient acceleration in models with holographic dark energy

Current literature usually considers the models where the required dynamics of Universe is provided by one or another, and always only one, type of dark energy. As was multiply mentioned above, in order to explain the observed dynamics of Universe, the action for gravitational field is commonly complemented, besides the conventional matter fields (both matter and baryon), by either the cosmological constant, which plays role of physical vacuum in SCM, or more complicated dynamical objects — scalar fields, K-essence and so on. In the context of holographic cosmology, the latter term is usually neglected, restricting to contribution of the boundary terms. Nevertheless such restriction has no theoretical motivation.

In the present subsection we consider the cosmological model which contains both volume and surface terms[178, 179]. The role of former is played by homogeneous scalar field in exponential potential, which interacts with dark matter. The boundary term responds to holographic dark energy in form of (281). Such model is shown to produce

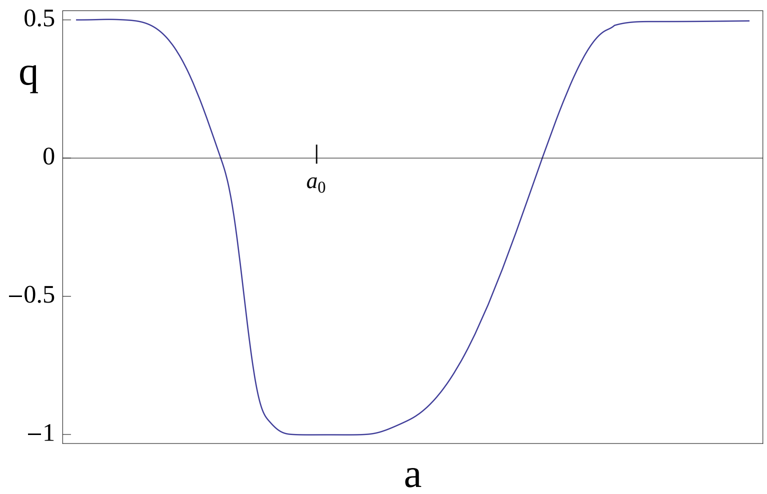

FIG. 13: The deceleration parameter dependence in some models with transient acceleration.

the transient acceleration phase — the evolution stage when the accelerated expansion of Universe changes to the decelerated one, and afterwards the Universe comes to the stage of eternal accelerated expansion, thus removing the limitations imposed in the previous subsection.

To describe the dynamical properties of the Universe it is convenient to to transform to dimensionless variables as the following:

$$x=\frac{\dot{\varphi}}{\sqrt{6}M_{Pl}H},\quad y=\frac{1}{M_{Pl}H}\sqrt{\frac{V(\varphi)}{3}},\quad z=\frac{1}{M_{Pl}H}\sqrt{\frac{\rho_m}{3}},\quad u=\frac{1}{M_{Pl}H}\sqrt{\frac{\rho_q}{3}}. \tag{331}$$

The evolution of scalar field is described by the Klein-Gordon equation, which in the case of interaction between the scalar field and matter takes the following form:

$$\ddot{\varphi}+3H\dot{\varphi}+\frac{dV}{d\varphi}=-\frac{Q}{\dot{\varphi}}. \tag{332}$$

In the present section we consider the case when the interaction parameter Q is a linear combination of energy density for scalar field and dark energy

$$Q=3H(\alpha\rho_\varphi+\beta\rho_m), \tag{333}$$

where α, β are constant parameter. For given model, regardless the explicit form of the scalar field potential $V(\varphi)$, the system of dynamical equations takes the following form

$$\begin{aligned} x' &= \frac{3x}{2}g(x,z,u)-3x+\sqrt{\frac{3}{2}}\lambda y^2-\gamma,\\ y' &= \frac{3y}{2}g(x,z,u)-\sqrt{\frac{3}{2}}\lambda xy,\\ z' &= \frac{3z}{2}g(x,z,u)-\frac{3}{2}z+\gamma\frac{x}{z},\\ u' &= \frac{3u}{2}g(x,z,u)-\frac{u^2}{n}, \end{aligned} \tag{334}$$

where

$$g(x,z,u)=2x^2+z^2+\frac{2}{3n}u^3,\ \lambda\equiv-\frac{1}{V}\frac{dV}{d\varphi}M_{Pl}. \tag{335}$$

and

$$\begin{aligned} Q &= 9H^3M_{Pl}^2\left[\alpha(x^2+y^2)+\beta z^3\right];\\ \gamma &= \frac{\alpha(x^2+y^2)+\beta z^3}{x}. \end{aligned} \tag{336}$$

As was mentioned above, here we consider the simplest case of exponential potential

$$V = V_0 \exp\left(\sqrt{\frac{2}{3}}\frac{\mu\varphi}{M_{Pl}}\right), \tag{337}$$

where μ is constant. Taking into account the expression (336), the system of equations (334) reads

$$\begin{aligned}
x' &= \frac{3x}{2}\left[g(x,z,u) - \frac{\alpha(x^2+y^2)+\beta z^2}{x^2}\right] - 3x - \mu y^2,\\
y' &= \frac{3y}{2}g(x,z,u) + \mu xy,\\
z' &= \frac{3z}{2}\left[g(x,z,u) + \frac{\alpha(x^2+y^2)+\beta z^2}{z^2}\right] - \frac{3}{2}z,\\
u' &= \frac{3u}{2}g(x,z,u) - \frac{u^2}{n}.
\end{aligned} \tag{338}$$

In the considered model the deceleration parameter takes the form

$$q = -1 + \frac{3}{2}\left[2x^2 + z^2 + \frac{2}{3n}u^3\right]. \tag{339}$$

Note that the cosmological parameters have no explicit dependence on the interaction form, and only determine the evolution dynamical variables. This fact essentially complicates analysis of such system. Let us make few remarks on the system of equations (338). First consider the case $y = 0$, which corresponds to free scalar field. It easy to obtain some restricting relations on the interaction parameters, which follow from the requirement for energy density to be real

$$2\sqrt{\frac{\beta}{\alpha}} > 1 + \alpha + \beta, \tag{340}$$

which in turn requires $0 > \beta > \alpha, |\alpha| + |\beta| < 1$. This unstable critical point corresponds to Universe filled by dark matter. The case of multiple such points is also possible but irrelevant. To conclude we note that any of such critical points is easily shown to exist also in the interval $x_0 < 0$. For the case $z \neq 0$ with the restrictions imposed by (340), one obtains

$$x_c = \left[\left(a + \sqrt{\beta/\alpha}\right)^{1/2} + a\right] z_c, \tag{341}$$

where $a = \left(2\sqrt{\beta/\alpha} - (1+\alpha+\beta)\right)^{1/4}$.

1. *Case $Q = 0$*

In the present section we consider in more details the interaction-free case with scalar field and dark matter. The critical points of the system (338) in the case ($\alpha = \beta = 0$) are given in the table I. The phase space generated by the system of equations (338) contains six physically relevant critical points, the last of which is an attractor. The first critical point $(1, 0, 0, 0)$ is unstable and corresponds to Universe dominated by scalar field with extremely rigid state equation ($w_\varphi = 1$), the second critical point is also unstable and responds to the evolution period when the scalar field is dynamically equivalent to cosmological constant.

The next third point $(0, 0, 1, 0)$ is of no interest because it corresponds to Universe composed solely of dark matter which is also unstable. The fourth critical point $(0, 0, 0, 1)$ corresponds to Universe solely composed of holographic dark energy in form of (281), and was considered above in details. The most physical interest is presented by the last sixth critical point which is an attractor. It corresponds to the Universe filled by scalar field and holographic dark energy. This critical point is completely determined by the scalar field potential parameter μ and magnitude of n :

$$\begin{aligned}
x_* &= \tfrac{2}{3n\mu}u_*, & y_* &= \sqrt{1 - \left(1 + \tfrac{4}{9n^2\mu^2}\right)u_*^2},\\
z_* &= 0, & u_* &= \tfrac{3}{2n\mu^2}\left(-1 + \sqrt{1 + \tfrac{4n^2\mu^4}{9}}\right).
\end{aligned} \tag{342}$$

TABLE I: Critical points for the autonomous system of equations (338)

(x_c, y_c, z_c, u_c) coordinates	Stability character	q	w_φ	w_{tot}
$(1,0,0,0)$	unstable	2	1	1
$(0,1,0,0)$	unstable	-1	-1	-1
$(0,0,1,0)$	unstable	$\frac{1}{2}$	$\nexists$	0
$(0,0,0,1)$	stable	$-1+\frac{1}{n}$	$\nexists$	$-1+\frac{2}{3n}$
$(-\frac{3}{2\mu}, \frac{3}{2\mu}, \sqrt{1-\frac{3}{2\mu^2}}, 0)$	unstable	$\frac{1}{2}$	$\nexists$	0
$(x_*, y_*, 0, u_*)$	attractor	$q_* < 0$	$w_{\varphi *}$	w_{tot*}

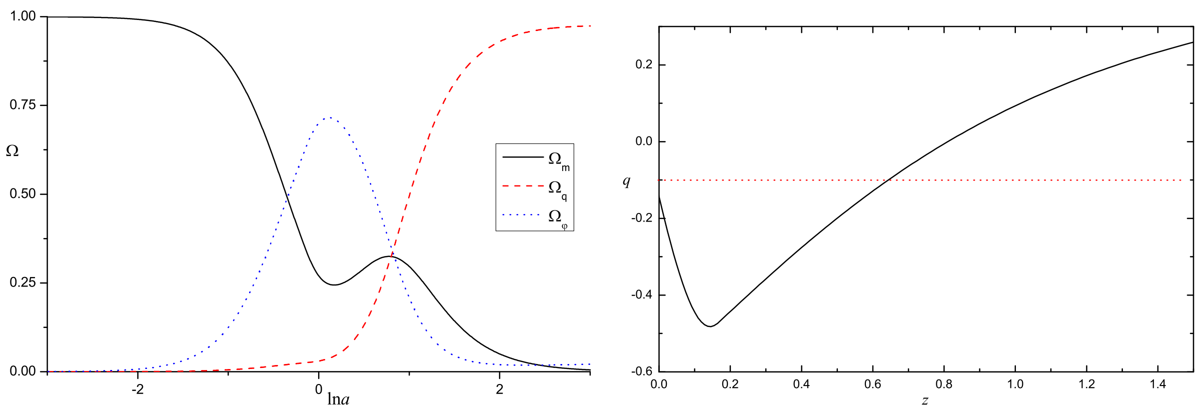

FIG. 14: Behavior of Ω_φ (dot line), Ω_q (dash line) and Ω_m (solid line) as a function of $N = \ln a$ for $n = 3$, $\alpha = 0.005$ and $\mu = -5$ (left side). Evolution of deceleration parameter for this model (right side).

The obtained property $x_* \propto u_*$ is typical for the so-called tracking solutions [201]. Remark also there is also so-called background interaction between the scalar field and dark energy, caused by the fact that the scalar field dynamics is affected by the holographic dark energy which has negative pressure and influences on the Universe expansion rate by means of Hubble parameter, which enters into the Klein-Gordon equation for the scalar field. In the attractor point the dark matter density turns to zero. To fit that model with the observations one has to strictly define the initial conditions such that the accelerated expansion of Universe starts before then the considered evolution phase begins.

2. *Review of the case* $Q = 3H\alpha\rho_\varphi$

In the above case, the phenomenon of transient acceleration that occurs in such a Universe does not match the observations. In order to fix that problem we consider a model where scalar field interacts with dark matter. In the present section we consider the case with the interaction parameter of the form (333) with $\beta = 0$. The Figure 14 shows the dependencies Ω_q, Ω_m and Ω_φ for the case when $\alpha = 0.005$, $\mu = -5$ and $n = 3$. From the explicit form of the equations and interaction character it is easy to see that neither type nor position of the above obtained critical points change when the interaction disappears. The latter affects only behavior of the dynamical variables, which corresponds to different trajectories in the phase space between the critical points. This corresponds to the fact that the interaction parameters enter only in the Hubble parameter derivatives of second order and higher with respect to time.

With such values of interaction parameters the transient acceleration begins almost in the current era. Like in the conventional cosmological models with dark energy in form of scalar field, the latter starts to dominate and causes the accelerated universe expansion phase. Along with the Universe expansion the contribution of Ω_q increases, which leads to the fact that the background (space) starts increasing faster than the field and becomes asymptotically free. Such field is known to have the so-called super-strict state equation and forces the Universe to decrease its expansion rate. Soon however, when the contribution Ω_q increases enough that the scalar field cannot any more prevent the expansion, Universe starts to speed up again.

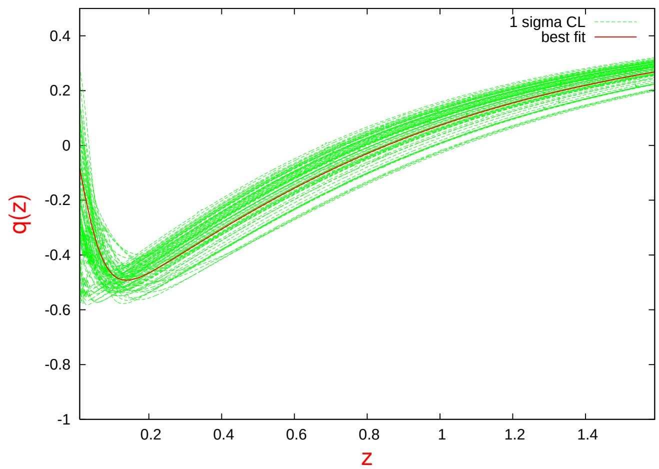

FIG. 15: The deceleration parameter dependence $q(z)$ reconstructed from independent observational data, including the brightness curves for SNe Ia, cosmic microwave background temperature anisotropy and baryon acoustic oscillations (BAO), SN and parametrization (344). The red solid line shows the best fit on the confidence level 1σ CL [102].

E. Observational evidence

Starobinsky [102] with co-authors, based on independent observational data, including the brightness curves for SNe Ia, cosmic microwave background temperature anisotropy and baryon acoustic oscillations (BAO), were able to show (see Fig.15), that the acceleration of Universe expansion reached its maximum value and now decreases. In terms of the deceleration parameter it means that the latter reached its minimum value and started to increase. Thus the main result of the analysis is the following: SCM is not unique though the simplest explanation of the observational data, and the accelerated expansion of Universe presently dominated by dark energy is just a transient phenomenon. The term "transient acceleration" is increasingly often used with respect to the Universe dynamics.

Remark, that the paper [102] also showed that with the use of CPL parametrization

$$w(z) = w_0 + \frac{w_a\, z}{1+z}. \tag{343}$$

for the state equation parameter, it is impossible avoid contradictions in attempt to combine the data obtained from observations of near supernovae of type SN1a with those of the relict radiation anisotropy. One of the way out of that contradiction is to reject the above mentioned parametrization and invent a new one. Starobinsky group suggested a novel parametrization which can combine those data sets:

$$w(z) = -\frac{1+\tanh\left[(z-z_t)\Delta\right]}{2}. \tag{344}$$

Within such approximation $w=-1$ on early times of Universe evolution and increases up to its maximum value $w\sim 0$ for small values of z.

Figure 15 shows the dependence of the deceleration parameter q reconstructed by the parametrization (344).

In the year 2010 in frames of Supernova Cosmology Project (SCP), the most recent data set on supernovae outbursts was published [103]. It contains 557 events and is largest up to date. Besides that the data set concerning supernovae with moderate red shift ($z<0.3$) was remarkably extended. Today there are several works [58, 59], making analysis of the above mentioned data in order to test the transient acceleration hypothesis. All the authors share common opinion that the ultimate answer will be given only by remade more accurate measurements. Moreover, obtain a convincing result one will probably have to remake all the data treatment procedure. Thus it was shown in [58, 59] that there is a contradiction between the data obtained from (SNe Ia+BAO) for small redshifts and those from (CMB) for high ones. The contradiction is in the fact that the analysis of two data sets separately one obtains opposite results. For example, using only (SNe Ia+BAO) data, one obtains evidence in support of the fact that the Universe expansion rate reached the maximum value at ($z\sim 0.3$) and presently starts to decrease. At the same time, if the data are complemented by the CMB data set, the results of analysis drastically change and no deviations from ΛCDM can be revealed. Therefore the result of reconstruction the evolutionary dependence of dark energy and answer to the question whether our Universe will expand with deceleration or accelerated expansion will continue forever (like in SCM), strongly depends on data obtained from SNe Ia observation, its quality, the particular method

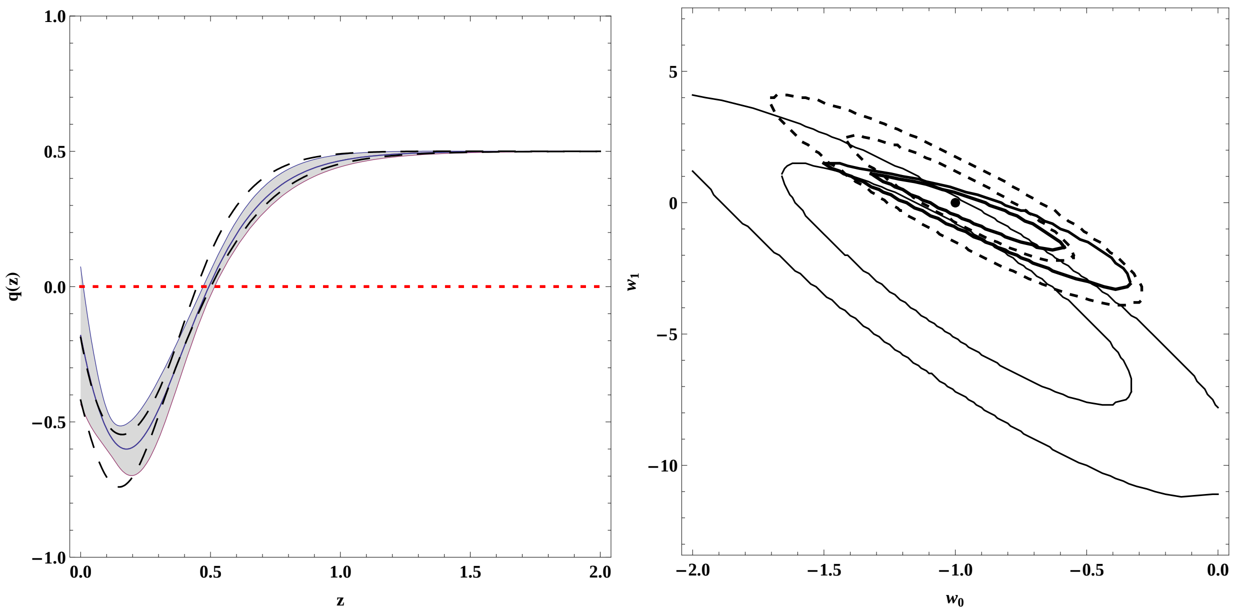

FIG. 16: On the left: the deceleration parameter dependence $q(z)$ reconstructed from results of analysis of Union2+BAO data sets with confidence level 2σ. The grey region and the one between the two dashed lines corresponds to presence and absence of systematic errors in the SNe Ia observations. On the right: the confident regions for 68.3% and 95% levels are shown for w_0 and w_1 in the parametrization CPL with $w = w_0 + w_1 z/(1+z)$. Dashed, solid, thin solid lines corresponds to the results of Union2S, Union2S+BAO and Union2S+BAO+CMB respectively. The point $w_0 = -1$, $w_1 = 0$ corresponds to spatially flat ΛCDM model(SCM).

of reconstruction for the cosmological parameters such as $q(z), w(z)$ and Ω_{DE}), as well as on explicit form of the state equation parametrization. For detailed answer to that question we have to wait more precise observational data and seek less model-dependent ways of their analysis.

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